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# A LINEARIZED, TIME DEPENDENT MODEL OF THE HEAT TRANSFER AND THERMOREGULATORY RESPONSES OCCURRING UPON IMMERSION IN COLD WATER

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## US ARMY RESEARCH INSTITUTE OF ENVIRONMENTAL MEDICINE
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much steeper slope with respect to the $T_s$ axis for small, thin men than for heavy, fat men, while men of average weight and fat composition exhibit an intermediate slope. Small, lean men also appear to exhibit thermogenic planes having steeper slopes with respect to the $T_{re}$ axis than do average and heavy men. The time course of skin and rectal temperatures as well as surface heat flow was stimulated for each individual immersion with the aid of a time dependent system of differential, heat balance equations coupling different body compartments and the epithelium to the water bath. Metabolic heat production for each immersion (supplied as functions of $T_{re}$ and $T_s$ minus 8% of the instantaneous value to account for respiratory losses) were used as heat source terms. This formulation permitted the evaluation of internal and external conductances as a function of subject morphology and water temperature. Analysis showed that both metabolic and cardiovascular compensation occurs at higher bath temperatures for small, lean men than heavy fat men. It also showed that body size (expressed by the ratio of mass to surface area) is more important than body fat content alone in determining maximal total internal insulation. An expanded model was used to predict the time course of skin and rectal temperatures as well assurface heat flow when external insulation is worn. The analysis showed that clothing can lower core temperature rather than elevate it, particularly for some heavy subjects by increasing internal conductances and lowering heat production. This occurs in the face of a net increase in external insulation.

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A Linearized, Time Dependent Model of the Heat Transfer and Thermoregulatory Responses Occurring Upon Immersion in Cold Water

by

Louis Strong, Ph.D. and Ralph F. Goldman, Ph.D.

US Army Research Institute of Environmental Medicine

Natick, MA 01760

Running Title: Predicting response to water immersion

Key words: Heat loss, water immersion; body size; fat; wet suits; insulation; vasoconstriction; thermogenesis.

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Abstract

Twenty male subjects (17 to 28 yrs of age) exhibiting a range of body weights ($60 \leq BW \leq 95$ Kg) and body fat ($7\% \leq BF \leq 23\%$) underwent total immersion while at rest in water between 36°C and 20°C. Their metabolic heat production, which were measured as functions of time and water temperature, were converted to explicit linear functions of core ($T_{re}$) and skin ($T_s$) temperatures for each individual immersion. These were used to define planes of thermogenic activity which allowed the comparison of the onset and magnitude of shivering between individuals of any morphological group. These thermogenic planes show a much steeper slope with respect to the $T_s$ axis for small, thin men than for heavy, fat men, while men of average weight and fat composition exhibit an intermediate slope. Small, lean men also appear to exhibit thermogenic planes having steeper slopes with respect to the $T_{re}$ axis than do average and heavy men.

The time course of skin and rectal temperatures as well as surface heat flow was simulated for each individual immersion with the aid of a time dependent system of differential, heat balance equations coupling different body compartments and the epithelium to the water bath. Metabolic heat production for each immersion (supplied as functions of $T_{re}$ and $T_s$ minus 8% of the instantaneous value to account for respiratory losses) were used as heat source terms. This formulation permitted the evaluation of internal and external conductances as a function of subject morphology and water temperature. Analysis showed that both metabolic and cardiovascular compensation occurs at higher bath temperatures for small, lean men than heavy fat men. It also showed that body size (expressed by the ratio of mass to surface area) is more important than body fat content alone in determining maximal total internal insulation. An expanded model was used to predict the time course of skin and rectal temperatures as well as surface heat flow when external insulation is

worn. The analysis showed that clothing can lower core temperature rather than elevate it, particularly for some heavy subjects by increasing internal conductances and lowering heat production. This occurs in the face of a net increase in external insulation.

## INTRODUCTION

Water immersion is widely recognized as a useful technique for studying the human response to both hot and cold stress because of the rapid changes in body compartment temperatures and in metabolic and cardiovascular compensation which can be accomplished as a result of the high thermal conductivity of the water medium (3,22,23,24,25). Several studies have sought to determine the surface heat conductivity for subjects at rest in water (11,12,22,24); reported values range from 40 to 200 kcal/h $^{o}C\ m^{2}$. The wide variation arises, for the most part, because of the different techniques used in measuring the very small thermal gradient between the skin and the thermal bath, and the different techniques used to determine surface heat flows. Partitional calorimetry has been applied to human immersion in cold water to determine core-skin conductance and surface heat transfer in the steady state (12,13,14,20,21,22). Few studies have used the transient behavior of core and skin temperatures to determine *internal* conductance values. Such an analysis is complicated by the difficulties in directly measuring instantaneous body heat stores. It is nevertheless useful, because it allows comparison between the internal conductance values determined under steady state conditions, where the physiological stresses are relatively mild, and the internal conductance values determined under transient state conditions, which obtain under a much greater range of physiological stresses. Knowledge of the time rate of change of heat stores within the body compartments also allows one to predict the time course of temperature changes within the body compartments for exposure times, ambient temperatures and stress levels which are not easy to study experimentally.

A number of studies have investigated the role of body fat in retarding rectal temperature decline (13,14,15,16,17,20,21). While subcutaneous fat provides significant insulation, body size itself can alter the maintenance of

thermal equilibrium. Carlson et al. (14) reported that body insulation varied directly with specific gravity, yet the fraction of body volume calculated to be involved in insulation was always greater than the estimated fat content. In a study of male subjects having widely varying body size (50 to 150 kg) and body composition (4 to 40% body fat), Buskirk and Kollias (20) obtained a regression equation for the total body insulation (I) in water at 15°C as a function of the mean thickness of subcutaneous fat (SF in mm) ($I = 0.117 + 0.033\ SF$, in h °C m$^2$/kcal). Rennie et al. (13) obtained a different maximal tissue insulation in both men and women exposed to 30°C water ($I = 0.05 + 0.024\ SF$). Kollias et al. (21) obtained still another expression for the total body insulation of a sample of American women in water at 20°C ($I = 0.086 + 0.021\ SF$). Kollias et al. did compare their data with that of men exposed to the same conditions and concluded that women conduct heat away faster than men with similar body fat content. The greater conductance of women was postulated to depend upon surface area and mass differences. Though the insulation component attributed to subcutaneous fat is similar in each study, the insulation component ascribed to body size is assumed to be constant for each population even though body size differed greatly between studies. Differences in body size and surface area were not considered in these regression analyses; the total insulation simply was considered one dimensional, as subcutaneous fat.

The effect of varying thermal mass is most apparent in the transient state of core temperature decline. Keatinge (17) reported that the rate of fall of rectal temperatures in water at 15°C increased in proportion to the inverse of mean skinfold thickness. Sloan and Keatinge (15) found that a similar relationship which held in 20°C water, could be improved by making allowance for the size of their subjects (as expressed by their body weight to surface area ratio BW/SA). These studies, unfortunately, did not allow for individual differences in metabolic heat production which influenced the rate of decline.

The present study applies a time dependent model of heat transfer to the problem of total body immersion in water at temperatures ranging from 35 to 20°C; we evaluate the internal and external conductances which are required to produce the time variable core and skin temperatures observed in a resting population having varying body size (60-95 kg) and body composition (7-23% body fat). To accomplish this, we solved a system of time dependent, coupled linear differential equations which represent the heat storage and heat flow between major body compartments. The thermal mass of the body core, subcutaneous fat and skin compartments were evaluated for each individual from his anthropometric data. The metabolic rates, which were measured as a function of time, were converted to explicit linear functions of core and skin temperatures for each individual immersion and were incorporated as heat source terms. This analysis permitted us to compare the effects of changing the total body insulation (via changes in vasoconstriction, body fat, body size) against the effects of metabolic heat production on stabilizing core temperature.

The literature reports wide variance in the metabolic heat resulting from shivering thermogenesis among subjects having very different body size and composition (13,16,20,21). Although the thermal onset for shivering does vary with body size and composition, the magnitude of heat production is also a function of the temperature of the thermal receptors which trigger the autonomic response. We have sought to examine the relationship between shivering thermogenesis, the temperature of certain body compartments and the morphology of the individual (body fat and mass and surface area). Least squares regression formulae for metabolic rates in terms of these temperatures were used to define planes of thermogenic activity in three dimensions; this allowed comparison of the onset and magnitude of shivering between individuals of any morphological group.

ETHODS

A series of studies was conducted with 20 healthy male subjects totally nmersed in water at 28 and 20°C; ten of these were also subjected to nmersion at 36, 32, and 24°C. The anthropometric data appear in Table 1. infold measurements were obtained at the biceps, triceps, subscapular and prailiac sites and body fat was calculated according to the method of Durnin nd Womersley (19). Each subject was tested at a selected water temperature, nce a week, over a period of ten weeks with two subjects studied each day, one the morning and one in the afternoon. The morning subject had no breakfast nd the afternoon subject no lunch and neither had exercised for at least an hour ior to the test. The immersion tank was 3 meters (ten feet) long, 3 meters (ten et) wide and 4.6 meters (fifteen feet) deep with a volume capacity of 7.8 thousand liters (ten thousand gallons). The water temperature was checked ach morning and afternoon and monitored continuously during the test; it was und to vary no more than ± 0.3°C on any given day. The water was thoroughly ixed prior to the test to attain a uniform temperature in the tank. However, uring the test the subject was in essentially "still" water. The ambient air mperature ranged from 23-24°C for the entire test. The subject was placed upine on a nylon mesh cot, attached to a platform that could be lowered with a oist into the water to a depth at which the water completely covered his head. he subject was asked to lie still except when he shivered. To keep him from oating, a lead strip was placed over his chest and one over his feet. ompressed air was bubbled through a water filled bottle and then to a Douglas ag from which the subject breathed using a J-valve with a mouthpiece. The ubject usually kept his eyes closed during the entire immersion period.

Core temperature was measured with a thermistor probe inserted 10 cm nto the rectum. The temperatures from 10 skin surface loci were measured om thermocouples fixed under a strip of thin surgical tape; this thin tape did

not change the temperature measured by the thermocouple on the surface of a heated copper man from that measured in its skin. The mean skin temperature was calculated according to the following weights: instep (0.05), calf (0.15), medial thigh (0.125), lateral thigh (0.125), back (0.125), chest (0.125), upper arm (0.07), lower arm (0.07), hand (0.06), and forehead (0.10). The rate of free convection from the skin surface was measured with heat flow sensors (RdF Corporation Microfoil P/N 20460) attached at five body surface sites, with the same surgical tape at the edges of the plate; the sensor area of the plate was left uncovered. The mean weighted heat flow was determined with the following weights: lateral neck (0.1), chest (0.3), triceps (0.2), lateral thigh (0.2), and calf (0.2). Quantification of the shivering response was determined by a single EMG electrode (Beckman) placed over the quadriceps of the thigh. Shivering was recorded both by continuous EMG tracing of this signal and by visual observation using a 5 point rating scale. Oxygen consumption and $CO_2$ production were measured every six minutes by collecting the expired air in a Tissot spirometer and analyzing for $O_2$ and $CO_2$ with a Beckman E-2 oxygen and a Beckman LB-2 $CO_2$ analyzer. Metabolic rates were calculated by the Weir formula (1). EKG and respiratory rates were continuously recorded on a Grass polygraph.

Ten of the subjects also donned three different types of wet suits and repeated the water trials at 20 and 28°C, while fully instrumented to collect skin and rectal temperatures and surface heat flows. The insulation values (I) of the wet suits in water were measured on an immersed copper manikin yielding the following:

polyurethane cell $I = 0.110$ h°C m$^2$/kcal = 0.61 clo

vinyl polymer $I = 0.067$ h°C m$^2$/kcal = 0.37 clo

neoprene foam $I = 0.133$ h°C m$^2$/kcal = 0.74 clo

These are the effective combined thermal resistances of the wet suits plus the trapped water layer. The resistance of each suit alone was assumed to be the

difference between the effective value and the insulation of the layer of still water surrounding an unclad manikin (I = 0.009 h°C $m^2$/kcal). The mass of water which was trapped between the subject and the suit was estimated from the volume capacity of the suit and the water displacement of the subject when immersed.

## DATA ANALYSIS METHODS

The core and skin temperatures from all subject-exposures were simulated with the aid of a time-dependent, multi-compartment model for one dimensional heat transfer, using input data consisting of skin and rectal temperatures, radial heat flow from the surface and the metabolic heat production. The model has similar features to the lumped parameter, one-dimensional versions developed by Stolwijk (2), Timball et al. (3) and Huckaba et al. (4), but it divides the heat storage of each compartment of a nude man undergoing whole body water immersion into three components: 1) core, 2) subcutaneous fat and 3) skin. This division has the advantage that it is easier to evaluate the mass of tissue within each compartment and at the same time it permits the determination of the role of body fat in accounting for the differences in individual responses. This formulation, furthermore, permits the separate evaluation of changes in internal and external conductances with water temperature. It is also easily expandable to treat the heat transfer between the additional component layers required when protective clothing is worn.

In the multi-compartment model, Figure 1, heat transfer occurs through N, not necessarily contiguous, compartments (N arbitrarily large) each of which includes a body core, a fat compartment, a skin compartment, and any external thermal protective (clothing, air and/or water) layers present. The heat transport equation balances the time rate of change of thermal energy stored within each compartment, with the net energy flux through each compartment

plus the energy added or subtracted due to sources or sinks operative within the compartment. If one ignores the energy flux through the thermal gradient within each compartment (i.e. requires each compartment to be isothermal), the time rate of change of energy stored per unit area in the i-th compartment is:

$$(m_i/A_i)\, c_i\, \dot{T}_i = \sum_{k,j \neq i} h_{ij}\,(T_j - T_i) + Q_i\,(T_i, T_k, \dot{T}_i, t) \qquad *$$

where $m_i/A_i$ is the mass of compartment i ($m_i$) divided by its surface area ($A_i$), $c_i$ is the specific heat capacity, and $h_{ij}$ is the heat transfer coefficient from compartment i to compartment j. The $Q_i$ represent active sources of thermal energy which may be linear functions of the temperatures $T_i$, $T_k$, $\dot{T}_i$, or time t. We have specified $Q_i$ for this study to comprise the metabolic heat production which is operative only in the core compartment. The defined system, system *, is presented as a linear system of coupled, first order differential equations, the solution to which may be obtained in closed form by taking a sum of exponential functions whose arguments are standard expansions of the eigenvalues of the characteristic equation for system *. A set of sufficient boundary conditions are the initial temperatures for each compartment. The initial temperature of the fat compartment was not measured but was arbitrarily set at the average of the initial skin and core temperatures. Following Stolwijk and Hardy (2), we have taken the pertinent thermal capacities as:

| | |
|---|---|
| $c_1 = 0.83$ kcal/$^{o}$C/kg | (core) |
| $c_2 = 0.70$ kcal/$^{o}$C/kg | (fat) |
| $c_3 = 0.80$ kcal/$^{o}$C/kg | (skin) |

The mass to area ratios for each compartment were determined from the anthropometric measurements on each subject, with the approximation that the

geometry for radial heat flow would be represented as a series of concentric right circular cylinders, each of length equal to the height of the test subject. The effective radius to the skin surface ($R_3$) was calculated from the equivalent cylinder having the same surface area as that determined by the DuBois formula (5). The thickness of the epithelium was assumed to be 0.0015 m (6). The surface area bounding the fat layer was calculated for the cylinder having a radius equal to ($R_3$ - 0.0015). The effective surface area of the core was calculated from the cylinder having the same volume as that given by the uniform mass: density ratio of the core. In these calculations the densities of visceral, fat, and epithelial tissues were taken as 1.1 g/cc, 0.918 g/cc, and 1.1 g/cc, respectively.

For each subject-exposure condition, a regression equation was obtained relating the metabolic heat production to a linear function of skin and rectal temperatures and the time rate of change of skin temperature. If the correlation coffecient was better than 0.8, the heat input ($Q_1$) in (*) was set equal to the linear regression equation, minus 8% of its current value to account for respiratory heat loss; otherwise $Q_1$ was set equal to the simple average metabolic rate (M) determined by the 10 measurements taken over the course of the one hour immersion minus the 8%.

In order to simulate the time course of the measured core and mean weighted skin temperatures of subjects wearing different types of wet suits, it was necessary to add two extra compartments of thermal protection. The space just proximal to the skin is a compartment where water rushes in and fills the space immediately upon immersion; thereafter this trapped water compartment becomes a stagnant pool having a fixed volume of water. The outer-most compartment is that of the protective wetsuit itself. The extra insulation afforded by these two additional compartments was determined from steady

state measurements on a copper manikin for three different wet suits. The specific heat capacities of these materials were taken as: (7)

$c_5 = 0.35$ kcal/°C/kg (polyurethane-cell)

$c_5 = 0.32$ kcal/°C/kg (vinyl polymer)

$c_5 = 0.37$ kcal/°C/kg (neoprene foam)

## RESULTS

### Heat Loss vs Water Temperature

The mean weighted skin temperature falls exponentially for all nude subjects, approaching in the asymptotic limit a temperature which is only slightly higher than the bath temperature (Figure 2). The mean steady state thermal gradient from skin to water bath for this subject population is $0.11 \pm .05$°C at 36°C; it is $0.4 \pm 0.3$°C at 32°C, $0.8 \pm 0.5$°C at 28°C, $1.0 \pm .5$°C at 24°C, and $1.49 \pm 0.6$°C at 20°C (Figure 2). The surface heat flow is greatest at 20°C, with the steady state heat flux from the head ($400 \pm 100$ kcal/m$^2$ h), and larger muscle masses of the abdomen ($300 \pm 100$ kcal/m$^2$ h), chest ($200 \pm 60$ kcal/m$^2$ h) and lateral thigh ($300 \pm 100$ kcal/m$^2$ h) exceeding that from the smaller, more distal masses such as triceps ($200 \pm 50$ kcal/m$^2$ h), calf ($175 \pm 10$ kcal/m$^2$ h) and instep ($50 \pm 20$ kcal/m$^2$ h) (Figure 3). While the skin heat losses and temperatures vary with position, a practical approach that permits the calculation of the total thermal imbalance of an individual is to use the concept of an area weighted mean heat flow. For all subjects who underwent immersion at a particular bath temperature, the area weighted steady state mean was $20 \pm 10$ kcal/m$^2$ h at 36°C, $90 \pm 10$ kcal/m$^2$ h at 32°C, $160 \pm 30$ kcal/m$^2$ h at 28°C, $210 \pm 50$ kcal/m$^2$ h at 24°C and $290 \pm 70$ kcal/m$^2$ h at 20°C; the relationship between area weighted mean heat flow and water temperature appears to be linear below the thermal neutral zone and averages 70 kcal/m$^2$ h for every 4°C drop in bath temperature (Figure 4).

The time course for the drop in mean weighted skin temperature for an individual may be closely represented as a summation of exponential functions, the major component of which decays with a time constant of approximately 2 minutes (Figure 2, lower right). Physically, this represents the loss of heat energy stored in the cutaneous layer. At the lowest bath temperatures, a second phase of temperature decline is observed, having a time constant of about 8 minutes (Figure 2, lower right). This second phase is representative of heat being abstracted from the underlying tissue and blood which dissipates through heating the skin. Cyclic perturbations occasionally superimpose upon the steady state skin temperature at the lower bath temperatures; these are caused by time associated, variable changes in the core to surface conductance (cyclic vasoconstriction).

Heat flow from the body surface immediately following immersion follows the same time course as does the fall in local skin temperature, since the free convective heat losses from the skin surface simply equal the difference between skin and water temperature times a constant, the coefficient of heat transfer from skin to water. Steady state heat flux measured at the extremities were usually attained following 10 min of immersion at any temperature. No measurable difference in the time course of heat flow from the head and trunk as compared to the extremities was noted (Figure 3), although the vasoconstrictive reduction of blood flow in the head and trunk may not occur as markedly and as rapidly as in the extremities.

Heat Loss vs. Shivering Intensity

Individual skin temperatures show a small variability between subjects - about 1$^{o}$C at the lowest bath temperature (Figure 2). One of the reasons for this variability is different shivering responses; local surface film conductance varies with the magnitude of agitation of the adjacent water layers. Figure 5 shows the variation of mean skin to water conductance with shivering intensity, for 10 Ss at the three lowest water temperatures. The surface conductance was

calculated from the mean weighted heat flow and the measured skin to water temperature gradient. A graded response of 5 arbitrary units (a.u.) of EMG activity corresponds to light shivering, 10 a.u. to moderate shivering, and 15 a.u., or above to violent shivering. Despite the scatter, it is clear that heat transfer increases monotonically with shivering intensity. Apparently, the range of the skin to water heat transfer coefficient ($h_3$) is between 150 and 300 kcal/m$^2$ h $^{o}$C for $20 \leq TW \leq 28^{o}$C, depending to a large degree on body motion.

## METABOLIC RESPONSES

In water at 35 and 32$^{o}$C, the oxygen consumption of all subjects is essentially the same as their pre-immersion values; about 12-15 l/h. In colder water, the $\dot{V}O_2$ increased to as much as 55-60 l/h. (Figure 6), i.e. the heat due to shivering amounted, on average, to between 4 and 5 times the resting metabolic rate. In general, metabolic rates increase as the water temperature decreases, until shivering exhaustion saturates the response. There is, however, marked individual variation at temperatures below 32$^{o}$C. The onset of shivering thermogenesis occurs, generally, at higher water temperatures for lean (< 70 kgm, < 12% BF) than for heavy subjects (> 90 kgm, > 20% BF) (Figure 6). While there are no clear statistical differences between the average metabolic heat output for the two groups during immersion at 28$^{o}$C, the lean group of subjects show a marked increase in heat production at 20$^{o}$C compared to the heavy subjects (Figure 6); the average metabolic increase shown by the lean subjects was 181 kcal/h compared to that of 46 kcal/h shown by the heavy subjects.

A subject's skin and rectal temperatures are crucial determinants of his metabolic response. In Figure 7, the variation of metabolic heat production is plotted against skin and rectal temperatures for five typical subjects whose body types may be characterized as: a) heavy and fat (BF = 0.23, BW = 96.24 kg), b)

average (BF = 0.11, BW = 84.72 kg; BF = 0.15, BW = 72.25 kg), or c) small and lean (BF = 0.07, BW = 69.33 kg; BF = .086, BW = 61.14 kg). The data presented were obtained after the initial cold shock following immersion, if any, had passed; i.e. when the skin temperature stabilized to a constant value. This restriction was made to simplify the analysis, as metabolic rates have been shown to depend also upon the rate of change of skin temperature (8). For any measured core and skin temperature combination, the metabolic heat production is larger for smaller, leaner subjects than for heavier, fatter ones. In the range of skin temperatures between 19-32$^{o}$C and rectal temperatures between 35-38$^{o}$C, the lean man almost always shivered more intensely than the heavy man, thus raising his metabolic heat production much higher.

Linear regression equations relating the metabolic heat production to skin ($T_s$) & rectal ($T_{re}$) temperatures were obtained for each subject from data collected for all immersion temperatures; these are plotted as three dimensional planes (Figure 7). The regression planes found for groups of different morphology show great differences of slope with respect to the $T_s$ axis. The thermogenic response of the average lean subject to a 1$^{o}$C change in mean skin temperature is 4.1 times the response of a fat subject. A subject of average build has a thermogenic plane that exhibits an intermediate slope with respect to $T_s$ (Figure 8).

Figure 8 shows the cumulative regression planes calculated for the entire subject pool, after it had been split into three categories: lean, with BW < 70 kg and BF < 12%; average, with 70 kg $\leq$ BW $\leq$ 90 kg and 12% $\leq$ BF $\leq$ 19%; and fat, with BW > 90 kg and BF > 19%. The three planes are defined by the following regression equations:

$$998 - 17\,T_s - 9.5\,T_{re}; \quad \text{(lean)} \quad r = .66$$

$$541 - 13\,T_s - 0.35\,T_{re}; \quad \text{(average)} \quad r = .75$$

$$-467 - 4\,T_s + 18.8\,T_{re}; \quad \text{(heavy)} \quad r = .44$$

Though there is considerable variability within each group, there are greater differences across the groups. The small and lean subjects show the greatest metabolic response to decrements in skin temperatures; the average subjects produced a thermogenic plane with smaller slopes with respect to the $T_s$ axis and an average elevation with respect to total thermogenic activity level which was intermediate between the lean and heavy groups. The heavy subjects, as a group, showed the smallest metabolic activity and the smallest change per decrement in skin temperature.

The thermogenic planes generally have a negative slope with respect to the $T_{re}$ axis; a positive slope may be found if shivering exhaustion occurs or if the depression of rectal temperature was not sufficient to drive shivering thermogenesis. The water temperature range was high enough that most lean and average subjects demonstrated a negative slope. The heavy subjects did not exhibit a rectal temperature depression greater than $0.5^{\circ}C$, while some lean subjects exhibited a rectal temperature depression of more than $2^{\circ}C$. Consequently, the range of validation of metabolic regressions upon rectal temperatures is not equivalent for the two groups of subjects. As a group, the heavy subjects showed a small average decrease in metabolic heat production with a drop in $T_{re}$.

## TIME COURSE OF SKIN AND RECTAL TEMPERATURES

To determine how the magnitude of the mismatch between the heat produced and the heat lost by the body may be used to evaluate internal conductances, a computer program was used to simulate the time course of the mean weighted skin and rectal temperatures of the twenty subjects for all exposures. Use of the program requires the experimentally determined metabolic heat production and also the approximate heat transfer coefficients from core to fat ($h_1$), from fat to skin ($h_2$), and from the skin to the infinite heat sink ($h_3$) presented by the water.

EVALUATION OF $h_3$

A computer fit to the mean skin temperature for each subject-exposure condition was obtained by varying $h_3$ until the experimentally observed steady-state temperature resulted. The steady-state temperatures are determined by $h_3$, as well as by the amount of heat added by conductive and convective transfer from other compartments. The characteristic time for the skin temperature decline depends upon both $h_3$ and the heat capacity of the skin layer. The $h_3$ values, which were determined from the area weighted heat flow measurements divided by the thermal gradient from skin to ambient (Figure 3), were used as input to the computer program; small corrections to this value were usually necessary in order to produce the actual experimental steady state average skin temperature and may represent, in part, corrections due to the shielding effects of the heat flow discs.

EVALUATION OF $h_2$

The layer of subcutaneous fat serves not only as additional resistance to heat flow, but also as an additional locus for heat storage. While it protracts the time development of the fall of rectal temperature, it may steepen the fall of skin temperature by effectively insulating the epithelial layer from its heat source. We make the simplifying assumption that it is the body layer which provides the fixed resistance to heat flow; i.e. that fat layer conductance is independent of blood flow variations (9). It is difficult to separate the insulation afforded by body fat from that afforded by the combined mass of fat and body core since the average gradient across the fat layer is not well defined nor measured. We have taken the view that $h_2$ across the subcutaneous layer is inversely proportional to the thickness of that layer. For a cylinder, this is approximately proportional to the ratio of fat mass to surface area. Hatfield and Pugh give the thermal conductivity per mm of subcutaneous fat as

175.68 kcal/$m^2$ h $^oC$ (10). If an average thickness of the subcutaneous fat layer is determined from skinfold thickness measurements, then the conductivity of an individual's subcutaneous fat layer ($h_2$) may be calculated from the formula:

$$h_2 = \frac{175.68}{(\text{bifold skinfold}-3)/2} \quad (\text{in kcal/m}^2\ \text{h}\ ^o\text{C})$$

where the skinfold is measured in mm (the double epithelial thickness is 3 mm) and assumed fixed for each subject; $h_2$ is used directly as input into the computer program.

EVALUATION OF $h_1$

For modeling purposes, the body core is considered as the source of all metabolic heat producing activity (basically in the liver and muscle tissues). It contains, as well, all circulatory pathways of convective heat transfer to the skin. In the present formulation, the convective pathways of heat exchange are in parallel configuration with the fixed resistances of fat and core structures. Since peripheral blood flow was not measured, however, there is no present advantage to preserve the identity of all separate components of core heat loss; accordingly, we have collected the variable peripheral and fixed core resistances into a single variable core resistance. A constant value of $h_1$ was determined for each subject-exposure condition by a trial and error procedure which yielded the best overall representation of the time evolution of rectal temperatures. Rectal temperature is a sensitive function of both a) the metabolic thermogenesis (M) and b) the sum of the internal and external thermal resistances ($1/h_1 + 1/h_2 + 1/h_3$). Our method of calculating the total tissue conductance requires no assumption that a steady state temperature distribution is achieved. The core temperature continues to drop long after the skin temperature has stabilized (Figures 9 & 10). This usually proves to be a vexing problem in calculating the tissue conductance, as it is difficult to determine instantaneous

heat stores. The success of the present formulation in accounting for body heat storage can be judged by the agreement between the calculated and the experimental temperatures (Figures 9 & 10).

## MODELING THE TIME DEPENDENCE OF CORE AND SKIN TEMPERATURES

Figures 9 and 10 present some representative skin and rectal temperatures as a function of time following immersion at 28°C and 20°C for six subjects of lean (BW < 70 kg; BF < 12%), average ($70 \leq BW \leq 90$ kg; $12 \leq BF \leq 19\%$), or heavy (BW > 90 kg; BF > 19%) body composition. The skin to water temperature gradients differ between subjects by as much as ± 0.5°C. This may be explained principally by differences in shivering intensity (which determine $h_3$) although core to skin conductances (i.e. $1/h_1 + 1/h_2$) play a minor role. Core temperature differences between subjects may be explained by differences in both thermogenic activity and intrinsic insulation ($1/h_1 + 1/h_2$). Although lean subjects demonstrate a much higher metabolic heat production than do heavy subjects, the extra heat produced by the lean subjects does not sustain as high a rectal temperature as the heavy subjects are able to maintain. By the end of a one hour immersion at 20°C, the rectal temperature difference between subjects in the three morphological groups is in excess of 1°C.

After one hour immersion, heavy men exhibited a higher core temperature at 20°C than at 28°C. This occurred, in part, because shivering thermogenesis is somewhat greater at the colder temperature, but primarily because peripheral tissue conductance is much smaller (by 0.48 to 0.7). The heavy subject does not maximally vasoconstrict at 28°C, as judged from the fact that his core to skin resistance is greater at 20°C. The lean subjects, on the other hand, generally demonstrated a lower rectal temperature in the colder water (see Figures 9 and 10). Computer fits to the rectal temperatures suggest that tissue conductances for lean subjects are fractionally greater at 20°C than at 28°C. The extra

metabolic heat production at the colder temperature does not fully compensate for the heat loss due to the larger thermal gradient; thus, the rectal temperature drops faster. One subject (Figure 10, center) did not maintain safe rectal temperatures even for an hour at $20^{o}C$, although his shivering heat production was high (400 kcal/h), because he needed a moderate cutaneous circulation to support shivering.

Rectal temperature profiles are more variable among subjects having average body fat ($12\% \leq BF \leq 19\%$) and body weight ($70 \leq BW \leq 90$ kg). Core temperatures do not uniformly drop faster in the colder water, because increases in metabolic heat production can compensate for the larger thermal gradient (Figure 9, center). Occasionally, a subject would exhibit initial rectal temperature increases, associated with an early onset of vasoconstriction plus an increased metabolic rate, reaching a maximum in about 10 minutes and dropping steadily thereafter. Other subjects would occasionally exhibit the opposite response; a delayed onset of vasoconstriction accompanied by an initial drop in rectal temperature, followed by a temporary recovery and then a steady decline (Figures 9, left & 10, left). In these cases a time varying $h_1$ is required to accurately simulate the transient core temperature behavior.

## THE DEPENDENCE OF CORE TO SKIN CONDUCTIVITY ON $T_w$

The total tissue conductivity from central core to skin surface depends upon body composition and skin temperature. Figure 11 shows the variation of total tissue conductance with the ratio of body weight/surface area and water temperature for the population of 20 Ss. The size of the subject population and its limited range of body types did not permit us to differentiate between the roles of lean versus fat body masses in controlling an individual's total internal conductance. Accordingly, the conductance data are presented as a function of total body weight/surface area, rather than as a function of lean or fat body

masses. Conductance is uniformly high and maximal for all subjects at 36°C. At lower water temperatures, the conductance data appear to divide into two major categories; subjects whose BW/SA quotients are less than about 40 kg/m$^2$ for this study generally exhibit minimal conductance values at water temperatures between 28-32°C. Below a water temperature of about 28°C, their circulatory heat losses increase because a greater perfusion of the muscle mass is necessary to support active shivering. The group of subjects whose BW/SA quotients are greater than 40 kg/m$^2$ on the whole demonstrate their minimal conductance at considerably lower water temperatures (20°C or less); in 20°C water, the heavy men show a 50-75% increase in whole body thermal resistance above that attained at 28°C, while the lean subjects show, on average, 30% less than at 28°C. Such differences can result in greatly varying endurance times from subject to subject. The combination of different shivering thresholds, different setpoints for the onset of vasoconstriction, and different lean and fat body masses, results in core to skin conductances which vary by a factor of three from lean to heavy subjects at 20°C (Figure 11).

## MODIFICATION OF SKIN AND RECTAL TEMPERATURES BY INSULATIVE CLOTHING

Figure 12 illustrates two examples of experimental and computer simulated rectal and skin temperatures as a function of time when external insulation is worn. The two subjects presented represent the lean and heavy body types undergoing immersion at 20°C, while nude and while wearing three different types of wetsuits. The major effect of the additional insulation upon skin temperatures is to raise the mean skin temperatures above the unclothed value, as if the effective water temperature were elevated. Similar surface temperatures as those obtained at 20°C with 0.37 clo insulation, would occur at 26°C without the extra insulation. Similarly, skin temperatures obtained while

wearing 0.61 clo of insulation would result if nude in water at 27.5°C, and those obtained while wearing 0.74 clo of insulation would result if nude in water at 29°C. Independent heat flow measurements indicate that the quasi-steady state mean body surface heat flow drops from 300 kcal/m$^2$h for the nude lean subject to 115 kcal/m$^2$h in the 0.74 clo wet suit (Figure 13b). On the other hand, mean weighted heat loss for the heavy subject only drops from 210 kcal/m$^2$h with no protective clothing to 160 kcal/m$^2$h in the 0.74 clo wet suit (Figure 13a).

Modeling estimates of the quasi steady state surface heat loss from these nude subjects fall between 20-50 kcal/m$^2$h lower than the measured values (Figure 13). The measured surface flux from nude subjects may be simulated by assuming a mean skin temperature which is 0.4°C higher than that measured. The measured surface flex is expected to be somewhat high because the heat flow discs add extra insulation to the underlying skin layer (Figure 13) and the measured values were not corrected for this extra insulation. It is significant that the time course of the computer simulated surface heat flow closely follows the time course of the measured heat flows for both heavy and lean subjects, even though the steady state flux differ by a small constant amount (20-50 kcal/m$^2$h). These differences diminish as one considers the surface flux from the same subjects wearing insulative clothing. This too is reasonable, since the perturbations of skin temperatures caused by the insulation of the discs are certain to be smaller with decreases in the thermal gradient from skin to surrounding layers.

The rectal temperature of a heavy subject is lower (0.3°C) after 60 min immersion when he is wearing a wet suit than when he is nude in water, while the opposite is true of a lean subject; his rectal temperature is 0.2°C higher when he is wearing a wet suit (Figure 12). While the metabolic rates for each subject drop with increasing amounts of insulation, modeling calculations suggest that the core to skin conductance exhibits two, quite different patterns as the

external insulation is increased. The core to skin conductance of the lean subject in all wetsuits is virtually identical to his conductance while nude in water at 28°C; this is the temperature at which his core to skin conductance is minimal. The heavy subject showed a higher conductance when clothed than when nude in the water. This behavior is in agreement with the finding that heavy subjects exhibit minimal conductance for skin temperatures less than 28°C. At $Tw = 20°C$, all of the heavy subjects in the study ($BW/SA > 40\ kg/m^2$) showed a higher core temperature when immersed nude than when wearing wetsuits. They maintain their minimum core to skin conductance without protective clothing and, indeed, the extra insulation appears to increase skin conductance in 20°C water; in contrast, lean subjects require additional insulation to achieve minimal conductance in 20°C water.

DISCUSSION

The purpose of this work is threefold: 1) to differentiate between those physiological responses to immersion in water at various temperatures which are constant and those which are variable in a sample population of adult males between 17-28 years of age; 2) to try to relate some of the individual differences to basic morphology differences; and 3) to test whether a relatively simple model of heat transfer can be used to incorporate different metabolic and vasoconstrictive responses in order to describe the time course of skin and rectal temperatures for a variety of subjects exhibiting a wide range of body fat and body weights. While metabolic heat production and the rate of rectal temperature decline exhibited great variability from subject to subject, skin temperatures showed a generally uniform dependence upon time and water temperature, with small variations (~1°C) in the steady state limit.

## Mean Weighted Skin Temperatures

Modeling calculations show that the biphasic fall of skin temperatures observed for bath temperatures less than 28°C may be interpreted as arising from two sources: 1) the loss of heat stored in the cutaneous layer which has a short time course (2 min) and, 2) the loss of heat added to the cutaneous layers from underlying layers where the rate constant is more protracted (8 min). The analytical solutions for the core and skin temperatures each involve a summation of exponential terms, the arguments of which depend upon the thermal diffusivity of different body components. The first order terms for the skin component involve the thermal diffusivity of the skin. The higher order terms, involving the thermal diffusivity of the underlying layers, become more important as the difference between the initial skin temperature and the bath temperature increases. Thus the biphasic nature of the exponential decline becomes more evident at 20°C than at 28°C.

Surface averaged heat transfer coefficients from skin to water ranged from 150 to 300 kcal/h $m^2$°C in this study; the variation depended on body motion. This range includes the values found by Witherspoon et al. (11) for copper manikin measurements in 'still water' and in water flowing at 0.15 m/sec., and the values found by Nadel et al. (12), for subjects at rest in still water (198 kcal/h $m^2$°C). It is, however, at least four times the value found by Boutelier et al. (22) who determined the convective heat exchange coefficient from the steady state metabolic heat production and the thermal gradient measured between the water bath and skin thermocouples covered by small copper plates and a permeable polystyrene tape.

The mean skin temperatures of heavy subjects (BW > 90 kg; BF > 20%) were not uniformly lower than those of lean subjects (BW < 70 kg; BF < 12%) in cold water ($T_w$ < 28°C). Analysis of the heat balance in each case showed that while lean subjects were maximally vasoconstricted in 28°C water, the heavy subjects

were not. Perhaps more important than differences in the thermal onset of vasoconstriction in determining mean weighted skin temperatures, however, were differences in body motion through shivering. Lean subjects always shivered more intensely than heavy subjects, thus raising their surface heat flow and reducing their mean weighted skin temperatures slightly.

The skin temperatures of subjects wearing protective wet suits exhibited a different time course than that of nude subjects, sometimes resolving into two turning points which result from the mass transfer of cold water into the layer underneath the garment; this layer subsequently absorbs body heat and becomes an additional layer of insulation. The computer simulations of the heat transfer from the skin surface to this insulative layer of water produced quasi steady state mean flux slightly lower (20-50 kcal/h $m^2$) than the measured values, yet consistent with the fact that measured flux are nominally higher than the actual values due to small increases in the skin temperature under the heat flow sensors.

Rectal Temperature Responses

The fall of rectal temperatures showed a highly variable time course because of measured differences in metabolic heat production and inferred differences in tissue conductances. The latter, in turn, result from differences in conductive and convective pathways in the subject population. Both fat and lean tissue mass differences alter conductive pathways. Convective pathways depend upon the extent of tissue vascularization, on the shunting of blood flow, and variations in countercurrent heat exchange. Shutdown of these convective pathways which accompany general vasoconstriction are triggered at different temperatures in different subjects. Satisfactory fits to the overall time course of rectal water temperatures could be achieved by utilizing only three constant, steady state heat transfer coefficients. Our approach shows that even though core temperatures are changing, the long term conductivity is operationally

constant. However, it is true that better fits may be obtained by allowing for variation in $h_1$ during the transitory phase of immersion.

Internal Conductances ($h_1$ and $h_2$)

The set of three coefficients which yielded the best representative curves of skin and rectal temperatures were reduced to two indicies of internal and external conductances. The usual methods for evaluating total internal conductance (the parallel sum of fat and core conductances) involve either a) taking the ratio of a measured surface heat flow and the core to skin temperature gradient or b) using the measured metabolic rate plus the assumed heat storage and the core to skin temperature gradient. The difficulty with the first approach is that perturbations of the surface heat flow are introduced by the presence of the sensors and an accurate surface weighted mean value heat flow is hard to obtain from a few spatially separated data points. In the second, errors can arise because of inaccurate assessments of instantaneous heat stores. Our evaluation of the instantaneous heat storage within the body core is also an estimate; any errors in that quantity would tend to be higher than the actual value since the entire body core is not uniformly at temperature $T_{re}$. This would cause our values of core conductances to be somewhat higher than the actual conductances. These errors diminish as the time rate of change of core temperature decreases and, for the rates of fall with which we are dealing, are usually negligible compared to the instantaneous heat flows.

More significant errors result when one makes the assumption of constant metabolic heat production. During the phase of quasi-constant surface flux, the metabolic rates (determined six minutes apart) could differ by 15% for successive intervals. Our formulation allows one to take account of the time varying metabolic rates which are often not considered in the evaluation of internal conductances, and to integrate the total heat inputs over time to obtain a more accurate determination of instantaneous heat stores.

## Fat vs Lean Body Insulation

Rennie et al. (13) compared the whole body insulation of Korean and American men and women when immersed in water at 30°C and 31°C, where it was apparently possible to achieve steady state core temperatures within 2-3 hours. They found average $1/h_1 + 1/h_2$ values of $0.109 \pm 0.01 m^2\,^oC$ h/kcal (N = 10) and $0.134 \pm 0.015 m^2\,^oC$ h/kcal (N=10) for American men and women respectively; comparable values for Korean men and women respectively were $0.113 \pm 0.007 m^2\,^oC$ h/kcal (N = 7) and $0.144 \pm 0.007\ m^2\,^oC$ h/kcal (N = 7). The differences within each group were attributed only to differences in the mean subcutaneous fat thicknesses. The differences between groups were attributed to the differences in the thickness of the non-fatty shell. Of the Americans, $0.05\ m^2\,^oC$ h/kcal was allocated to the insulation of this component; the remainder was assigned to the subcutaneous fat. For the Koreans, $0.092\ m^2\,^oC$ h/kcal was allocated to the insulation of their extrapolated lean body mass. Carlson et al. (14) calculated that the insulation for a group of subjects undergoing immersion in whom body fat constituted 8%-32% of the body weight varied directly with specific gravity. A variation of $1/_{h_1} + 1/_{h_2}$ values between $0.1\ m^2\,^oC$ h/kcal and $0.4\ m^2\,^oC$ h/kcal, produced a linear plot of maximum body insulation versus percent body fat which extrapolates to a lean body mass insulation less than $0.01\ m^2\,^oC$ h/kcal.

Sloan and Keatinge (15) measured the rates of body cooling of some 28 subjects, aged 8-20, whose subscapular skinfold thicknesses ranged from 5.0 mm to 19.2 mm, and whose weight ranged between 21 kg and 82 kg. They found that the rate of fall of sublingual temperatures increased as body size decreased, and correlated with the product of mean reciprocal fat thickness and the surface area to mass ratio of the subjects (r = 0.91).

In view of the relatively small insulation values measured directly for isolated slabs of subcutaneous fat (10), it is difficult to see the physical

justification for attributing far greater insulation to the total in situ fat mass than to an equivalent thickness of isolated fat. The probable explanation for the success of regression equations which provide that 1 mm of subcutaneous fat can have five to ten times the insulative value of a slab of non-vascularized tissue of equivalent thickness (0.0057 $m^2$ °C h/kcal) is that first, a limited range of body sizes are represented in the frequency distribution used to determine the regression equation and, second, that these distributions were considered to be only one dimensional in subcutaneous fat (SF) rather than two dimensional in SF and lean body mass/SA. Further, most of these studies considered only a single bath temperature where the full range of an individual's circulatory compensation was not in evidence. In the present model, the insulation values attributed to the subcutaneous fat mass were adjusted to 0.0057 $m^2$ °C h/kcal per 1 mm of skinfold thickness.

<u>Circulatory Losses</u>

Although circulatory heat losses were not measured in this present series of experiments, their changes with water temperature could be inferred from changes in the total internal insulation. Figure 11 shows that the setpoint for the onset of vasoconstriction differs with subject morphology. In agreement with the results of Cannon and Keatinge (16), we find that maximal tissue insulation was achieved at a significantly higher bath temperature for small, lean subjects (BW/SA < 40 kg/$m^2$) than for large, heavy subjects.The variation of a man's total internal insulation (defined by $1/_{h_1} + 1/_{h_2}$ in the present model) is ascribed entirely to circulatory compensation rather than to modification of the intrinsic resistance of the fat layer which is completely passive. This compensation amounts to an average 100% increase in the small man's total internal insulation when the bath temperature drops from 36°C to 28°C (Figure 11); it amounts to an average 40% increase in the heavy man's total internal insulation when the bath temperature drops from 28°C to 20°C. Considering this

swing of possible conductances, we see that convective pathways are more effective than conductive pathways in the removal of heat; and further, when maximally vasoconstricted the core mass can account for greater insulation than the fat mass.

Shivering Thermogenesis

It has been reported that the metabolic heat production rises faster in thin than in fat men when the bath temperature is lowered below 33°C (16,17); our results confirm this finding. We have further attempted to relate shivering thermogenic activity of the different morphological types to their core and skin temperatures. Though the rectal and skin sites are but two of the many sites whose sensory inputs go into the determination of the total integrated thermoregulatory response (18), they appear to be a satisfactory number both for highlighting the individual differences in thermogenic activity, and for determining its temporal dependence in the the context of the present model. The thermogenic planes which describe the variation of metabolic heat production as a linear function of skin and rectal temperatures have clearly distinguishable slopes with respect to the $T_s$ axis for heavy and lean subjects. It would appear that the cold stimulation of surface thermosensitive neurons produces a higher level of autonomic motor activity in lean men than in heavy men, in the temperature range between 20-30°C. This has the effect of reducing the width of their thermoneutral zone from the low temperature side. Possibly a learned adaptation, the early onset increase in metabolic heat production serves to lengthen their survival time in cold water. If lean men were not as responsive to surface heat loss, their core temperatures would fall much faster than is the case. Yet even with their sizable outputs of metabolic power the lean cannot maintain nearly as high core temperatures as heavy subjects. Such differences may determine which individuals can benefit from "vasoconstrictive regulation" in the cold and which ones can only adopt "metabolic regulation".

The thermogenic planes respresenting the three body types have small slopes with respect to the $T_{re}$ axis; this appears to be in contradiction with the dramatic thermogenesis some individuals show during rectal temperature drops. This is a statistical manifestation of the high variability among the subjects' core temperatures for a given value of M, and apparently of differences in the set point for shivering. Core temperatures are functions of the total heat stores which were created by previous shivering which was both peripherally and centrally elicited. Differences in the slopes of the thermogenic planes with respect to the $T_{re}$ axis are not so obviously differentiated by subject morphology in this study. One difficulty in ascertaining morphological dependence is that the heavy subjects did not experience the severe depressions of core temperatures, that might induce equivalently high metabolic activity, as the lean subjects did. Further work involving the immersion of heavy subjects at lower bath temperatures is needed to elucidate these differences.

Finally, we have shown that the extra insulation afforded by wearing thermally protective garments has a calculatable effect on the time course of skin temperatures (Figure 12) and surface heat flows (Figure 13). Its effect upon metabolic heat production and the changes in body conductance are subtle and lead to some interesting alterations of core temperatures. For example, the simulation of core temperatures of fat, heavy subjects clearly show a degradation in whole body insulation when they are outfitted in protective garments compared to when they are nude. Paradoxically, the rectal temperature drops faster, even though the total insulation (body plus garment) is as great or greater than the maximum nude insulation. This paradox is resolved by the realization that metabolic heat production is also reduced. The use of protective garments may afford, as in this case, an insidious sense of security. A heavy subjects' skin temperature is elevated, yet his core temperature drops faster than it otherwise would. Presumably, at a sufficiently low core

temperature, the signal to increase autonomic motor activity, and to decrease convective (circulatory) pathways, will be sent down the neural axis to the appropriate effectors. More work is needed to examine the role of deep body receptors in eliciting the action of autonomic effectors.

TABLE I

ANTHROPOMETRIC RECORD OF SUBJECTS PARTICIPATING IN THIS STUDY

| Subject | Body Weight (kg) | Height (m) | Mean Skinfold (mm) | Body Fat (%) | Surface Area ($m^2$) |
|---|---|---|---|---|---|
| 1. | 61.14 | 1.722 | 5.28 | 8.6 | 1.72 |
| 2. | 68.09 | 1.679 | 8.42 | 14.1 | 1.78 |
| 3. | 61.26 | 1.703 | 6.32 | 10.7 | 1.71 |
| 4. | 84.72 | 1.816 | 6.48 | 11.0 | 2.06 |
| 5. | 69.33 | 1.752 | 4.73 | 7.3 | 1.84 |
| 6. | 64.35 | 1.74 | 5.33 | 8.7 | 1.77 |
| 7. | 70.07 | 1.688 | 5.75 | 9.6 | 1.80 |
| 8. | 73.99 | 1.765 | 8.85 | 14.7 | 1.90 |
| 9. | 66.89 | 1.778 | 4.89 | 7.7 | 1.83 |
| 10. | 64.43 | 1.737 | 6.11 | 10.3 | 1.78 |
| 11. | 96.24 | 1.856 | 16.93 | 16.9 | 1.86 |
| 12. | 70.22 | 1.768 | 10.63 | 16.9 | 1.86 |
| 13. | 72.12 | 1.833 | 8.70 | 14.5 | 1.94 |
| 14. | 95.55 | 1.78 | 17.63 | 23.1 | 2.10 |
| 15. | 63.40 | 1.762 | 6.88 | 11.7 | 1.78 |
| 16. | 66.35 | 1.762 | 6.54 | 11.1 | 1.82 |
| 17. | 69.13 | 1.703 | 9.15 | 15.1 | 1.80 |
| 18. | 72.25 | 1.745 | 9.94 | 16.1 | 1.87 |
| 19. | 63.51 | 1.689 | 7.17 | 12.2 | 1.73 |
| 20. | 73.34 | 1.710 | 12.63 | 19.0 | 1.85 |

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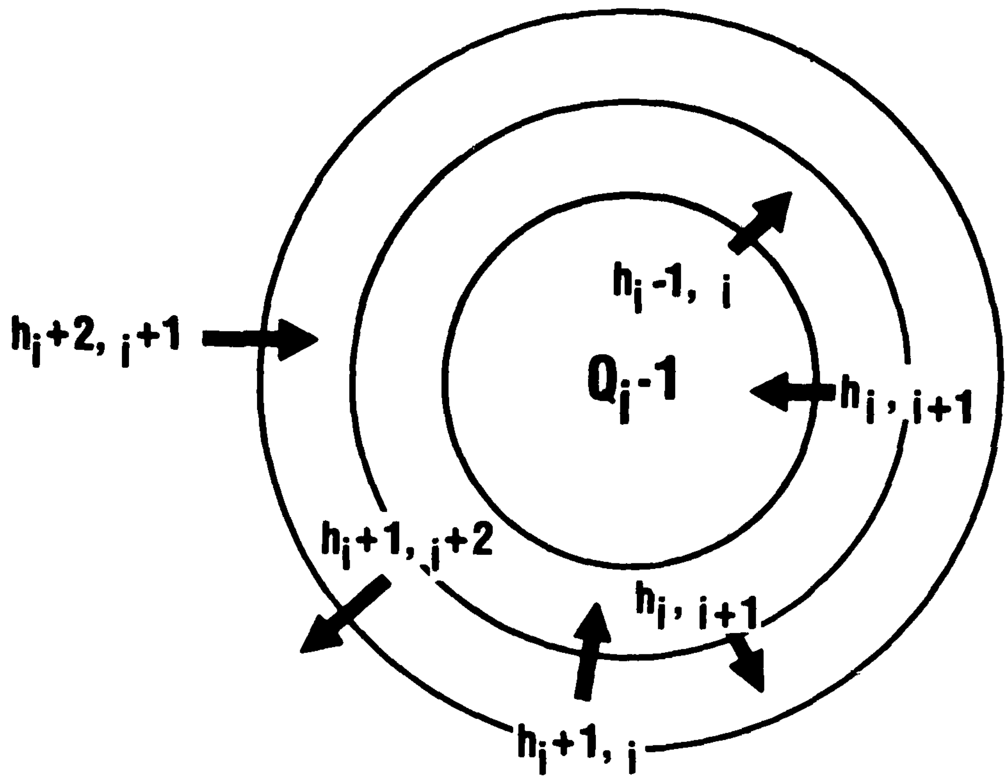



$$c_i m_i / A_i \dot{T}_i = \sum_{j,\, k \neq i} h_{i,j} (T_j - T_i) + Q_i (T_k, T_i, \dot{T}_i, t)$$

**FIGURE 1**

Figure 1 Diagram of linearized model for radial heat transfer through N compartments which interact via an NxN array of heat transfer coefficients $h_{i,j}$. A mass $m_i$, surface area $A_i$, specific heat $c_i$, and a source of heat production or extraction $Q_i$, is allocated to each compartment i.

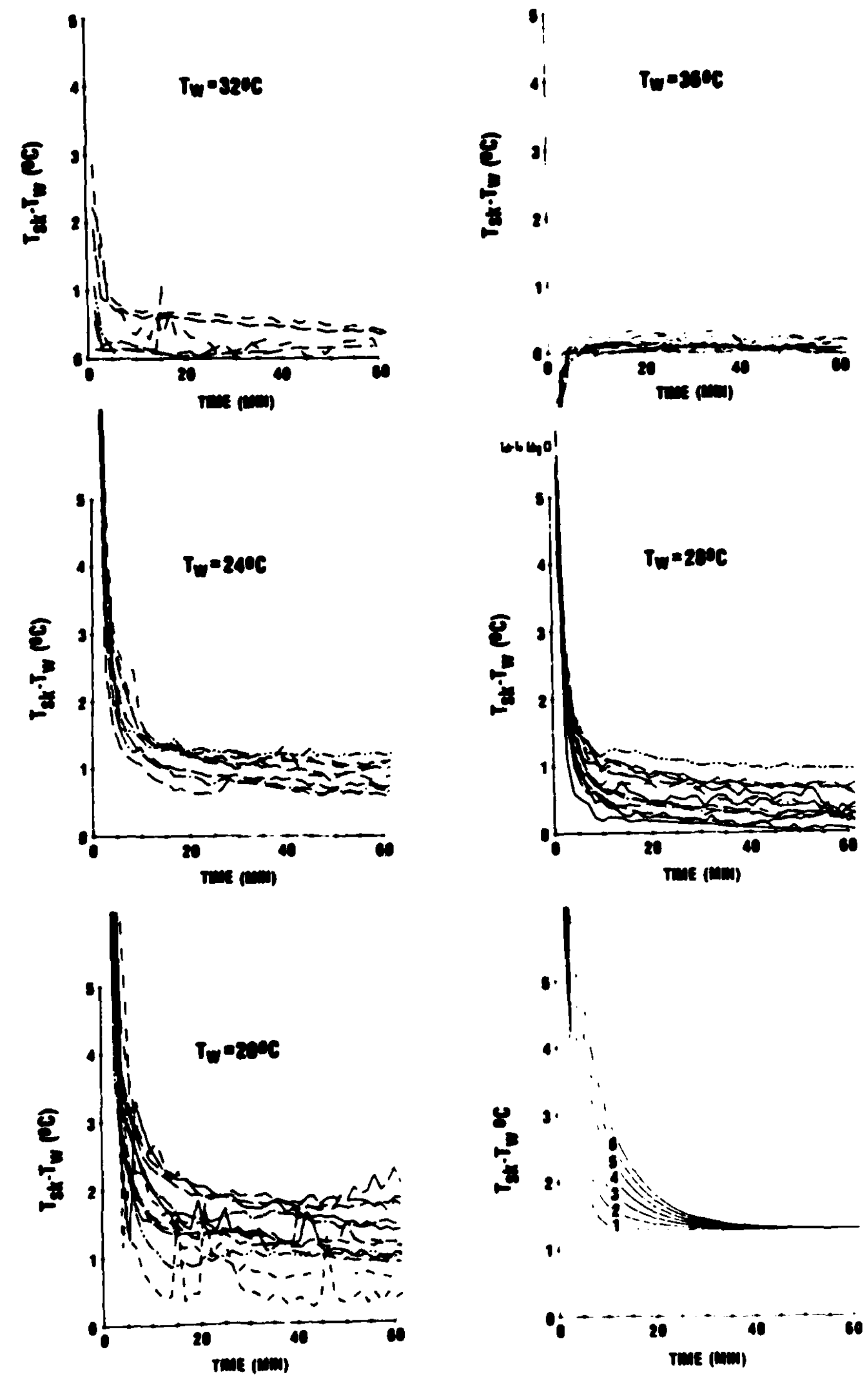



Figure 2 The time course of the average area weighted mean skin temperatures, for a group of nude subjects undergoing whole body water immersion at 20°, 24°, 28°, 32°, and 36°C. Skin temperatures achieve quasi-steady state values within 15 minutes of immersion. During immersion at 20°C, a biphasic temperature decay is observed, having time constants of 2 min and 8 min. In the lower right figure, the 8 min component is added according to the following normalized weights 1, 0; 2, 0.085; 3, 0.171; 4, 0.256; 5, 0.342; 6, 0.427.

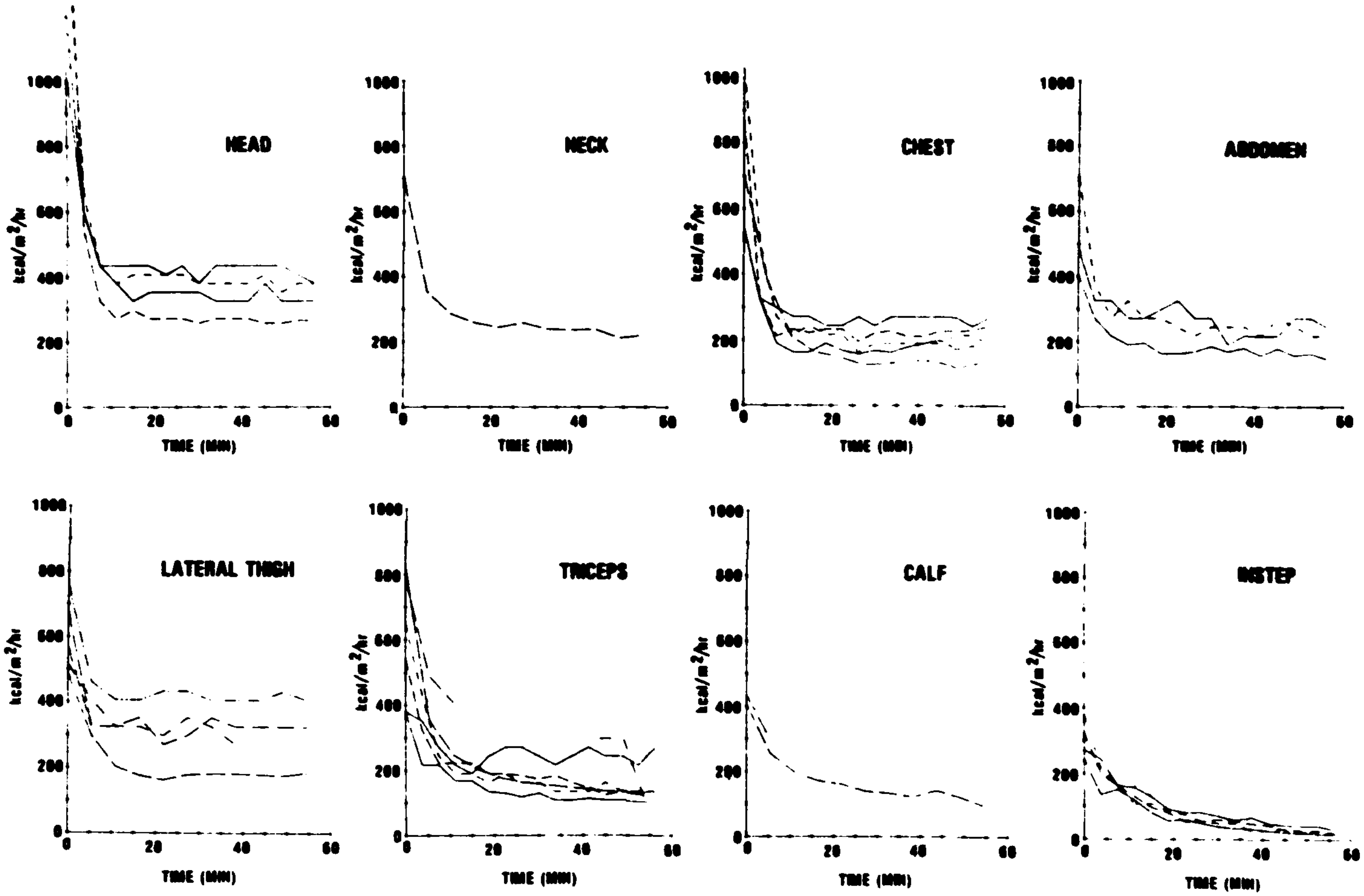



Figure 3 The anisotropy of surface heat flow determined at the following sites: lateral neck or forehead, chest, abdomen, lateral thigh, triceps, calf and instep for subjects in water at $20^{o}C$

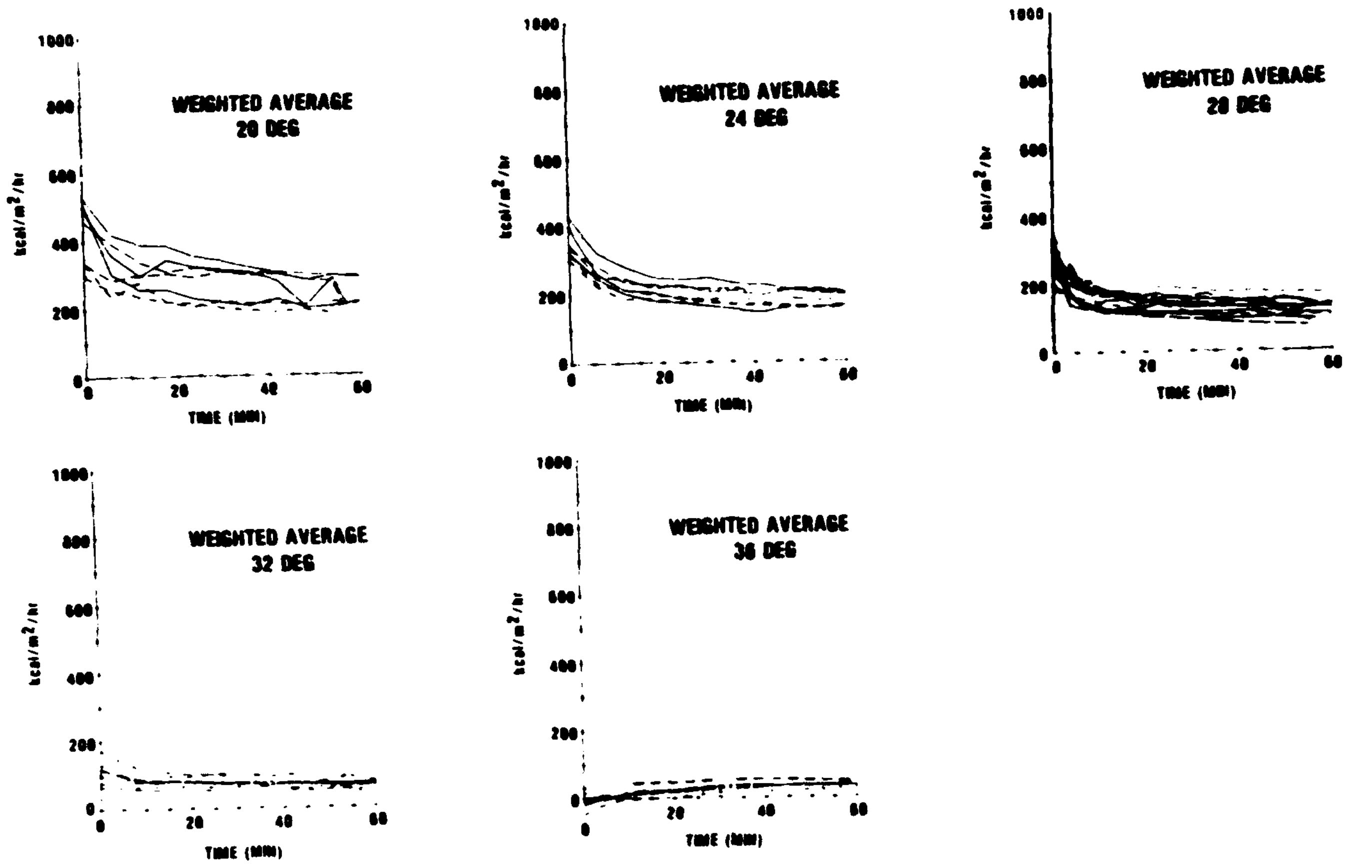



Figure 4 The time course of area weighted mean surface heat flow determined for subjects in water at 20°, 24°, 28°, 32° and 36°C.

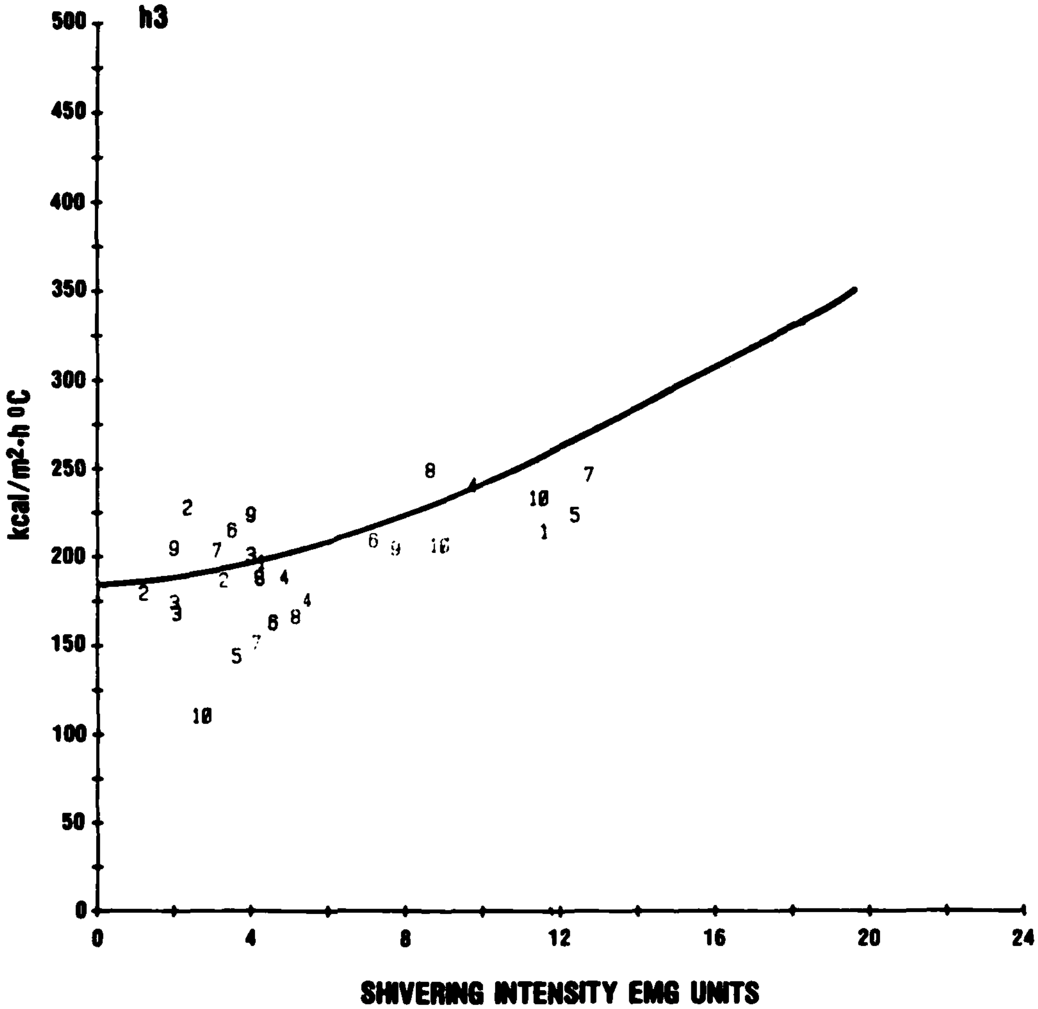



FIGURE 5

Figure 5 Skin conductance as determined from the quotient of surface heat flow and the thermal gradient from skin to water. Shivering intensity was measured at 20°, 24°, and 28°C with an EMG electrode affixed to the quadriceps of the thigh. Numbers indicate the subjects for whom the surface conductance and shivering intensity were determined.

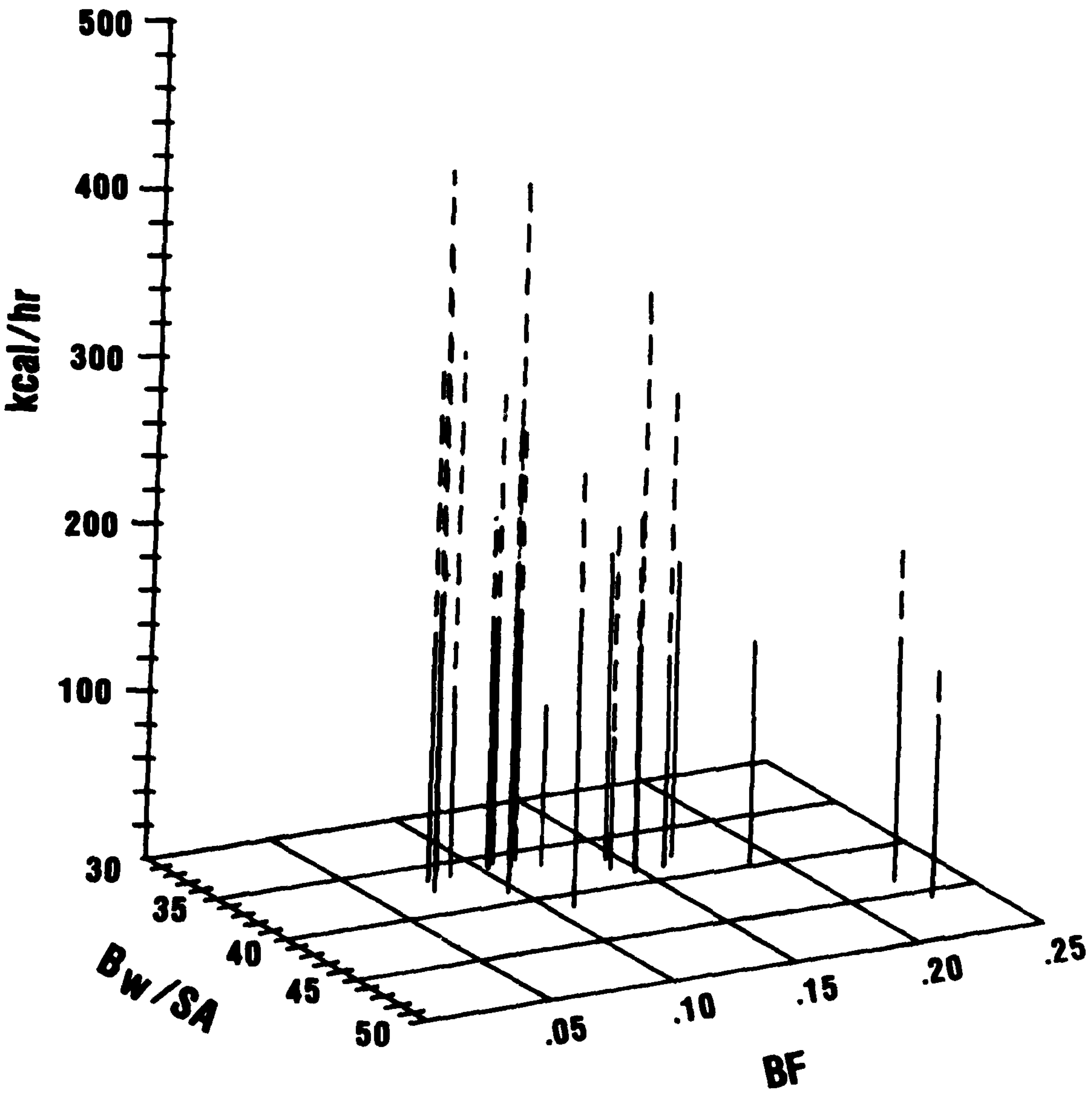



Figure 6 The mean rate of heat production determined during the last 50 min of immersion at 28°C (——) and 20°C (----) for each subject, plotted as a function of his body weight and body fat.

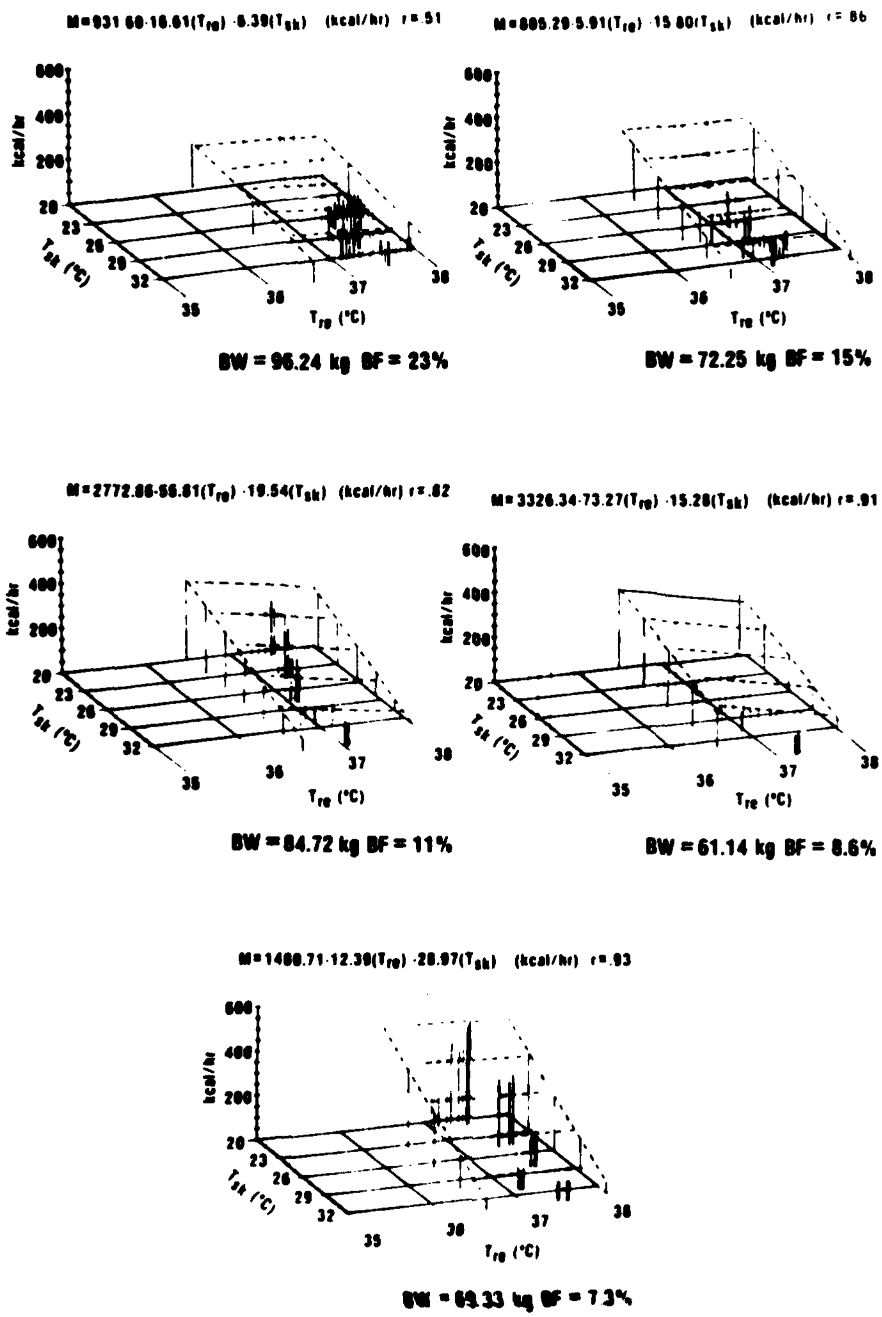
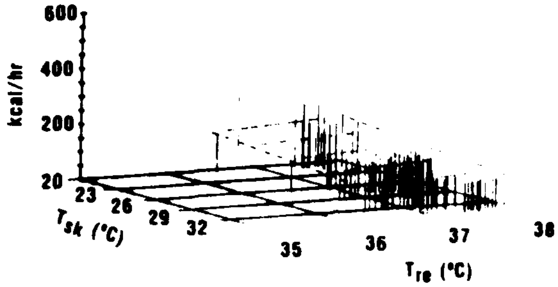
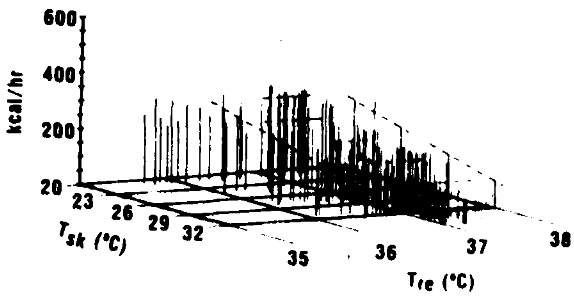


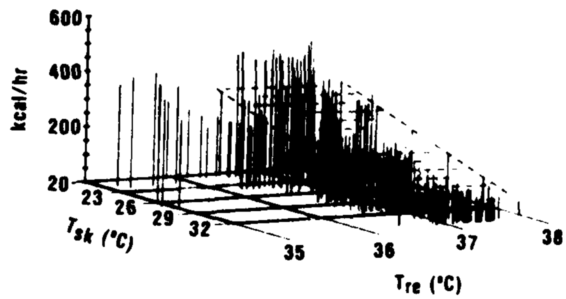

Figure 7 The rate of metabolic heat production, plotted as a function of both skin and rectal temperatures for subjects representing small, lean; average; and heavy, fat body types. The thermogenic planes, which are defined by the regression equations shown for each subject, indicate the individual variability. Dashed lines upon the plane indicate the slopes with respect to the $T_{re}$ and $T_{sk}$ axes; adjacent vertical lines represent a 50 kcal/h incremental rise. Extension of the planes was limited by the range of the data collected.




**HEAVY FAT SUBJECT GROUP (BW>90kg; BF>20%)**




**AVERAGE SUBJECT GROUP (70kg<BW<90kg; 12%<BF<20%)**




**SMALL, LEAN SUBJECT GROUP (BW<70kg; BF<12%)**

**FIGURE 8**

Figure 8 The three planes which represent the best statistical prediction of metabolic rates (M, in kcal/hr) as linear functions of skin and rectal temperatures for the three body types. They are defined by the following regression equations:

$$M = 998 - 9.5\,T_{re} - 17.3\,T_{sk} \;;\quad \text{(small, lean)} \quad r = 0.66$$

$$541 - 0.35\,T_{re} - 13.2\,T_{sk} \;;\quad \text{(average)} \quad r = 0.75$$

$$-467 + 18.85\,T_{re} - 4.2\,T_{sk} \;;\quad \text{(fat, heavy)} \quad r = 0.44$$

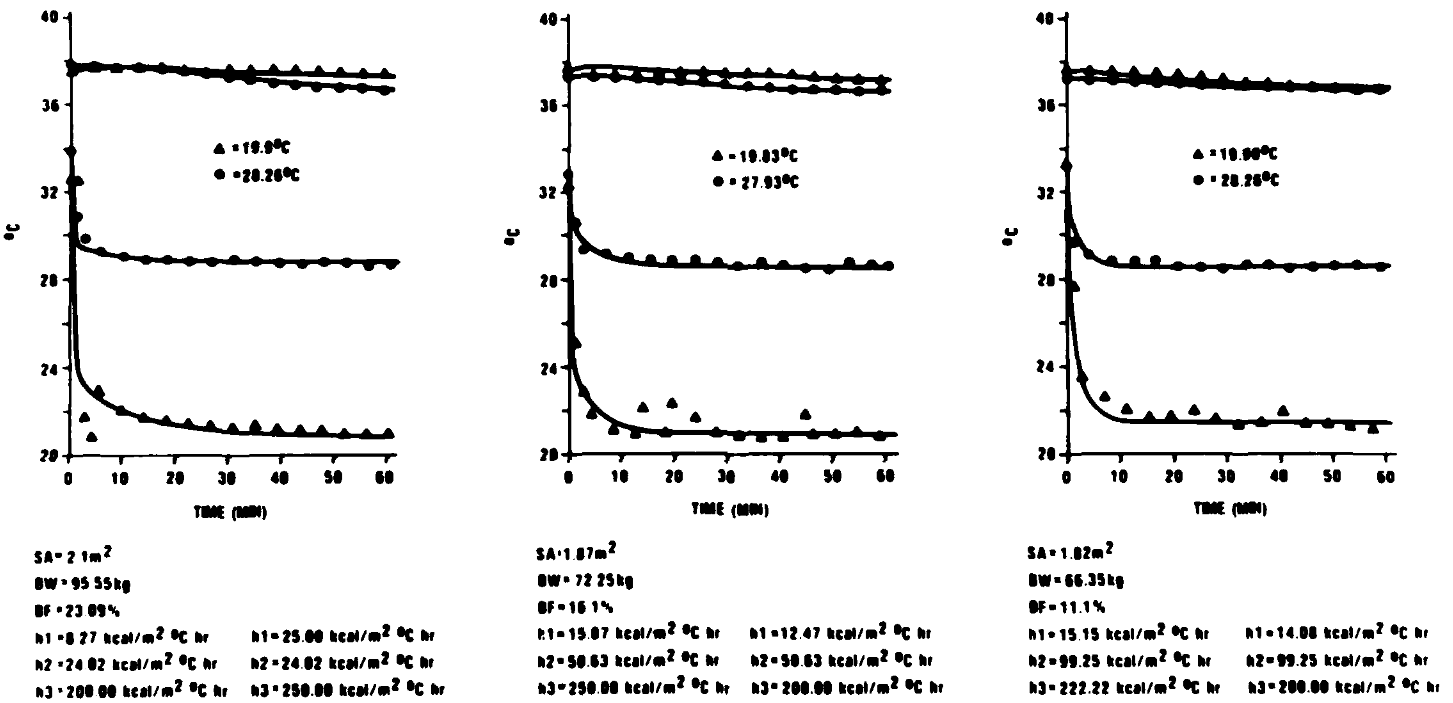



Figure 9 Experimental versus computed time course of skin and rectal temperatures for representative subjects undergoing immersion at 20°C and 28°C; o = measured skin and rectal temperatures for 28°C immersion and Δ = measured skin and rectal temperatures for 20°C immersion. Solid lines are computed temperatures. Left to right: subject 14, subject 18, subject 16. The data were simulated using constant heat transfer coefficients for the entire exposure.

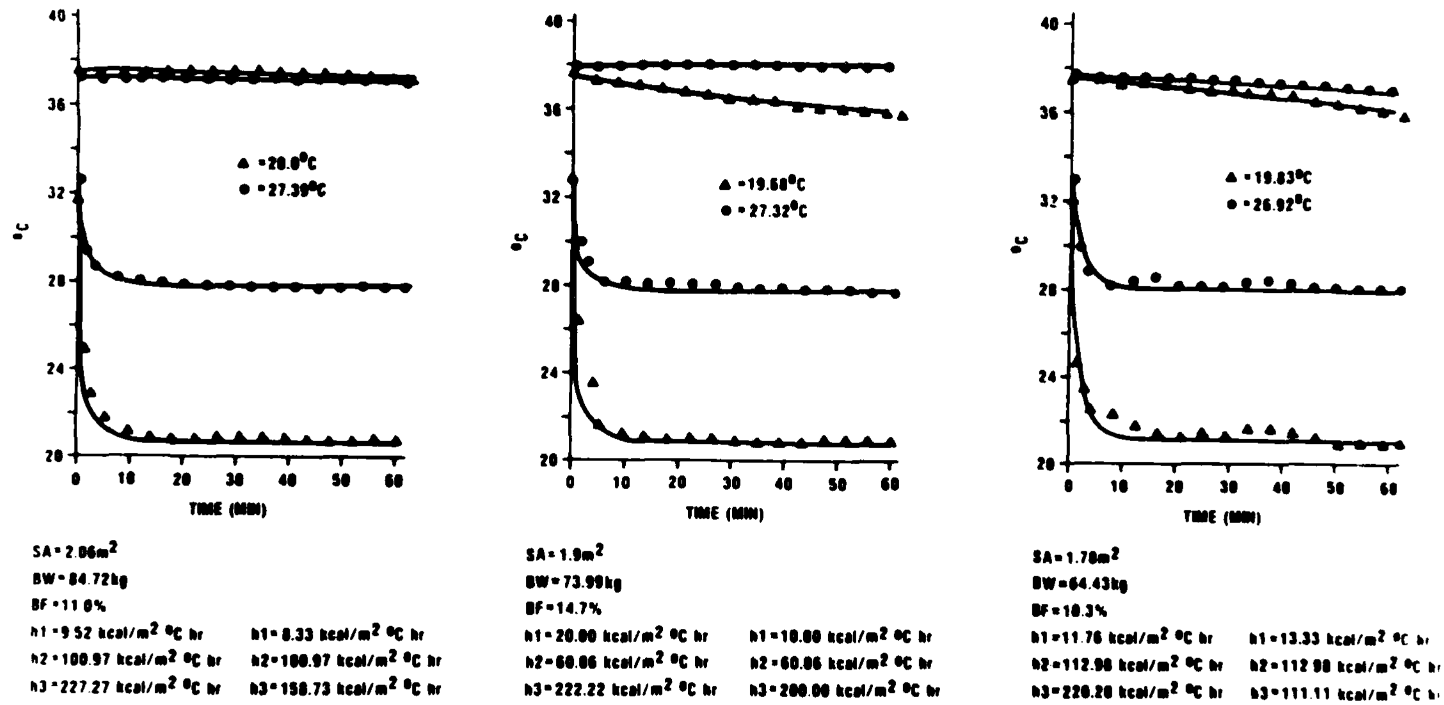



Figure 10 The time course of experimental and computed skin and rectal temperatures for representative subjects undergoing immersion at 20°C and 28°C; o = measured skin and rectal temperatures for 28°C immersion; Δ = measured skin and rectal temperatures for 20°C immersion. Solid lines are computed temperatures; Left to right: subject 4, subject 8, subject 10.

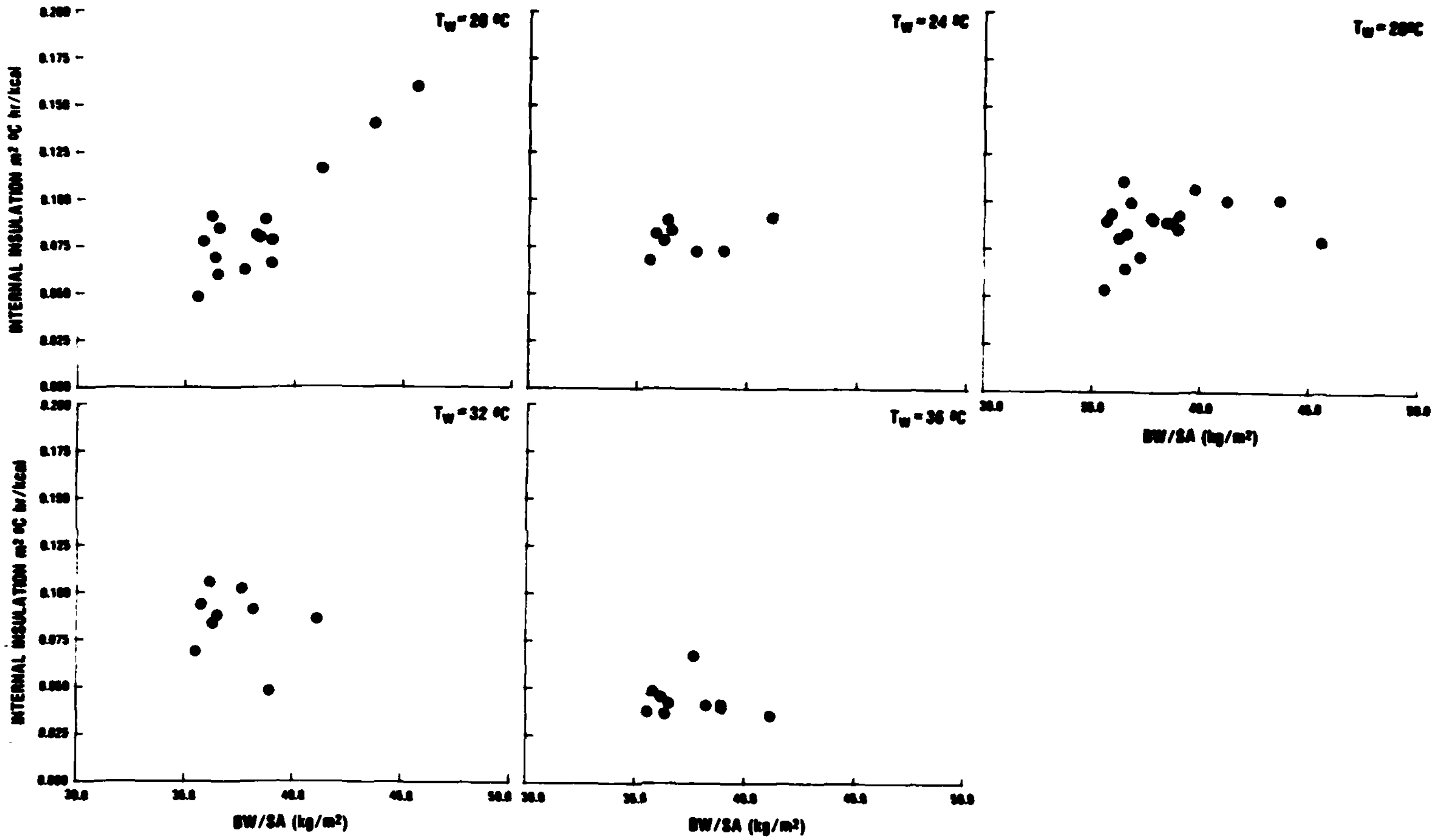



Figure 11 The variation of internal insulation versus body weight/surface area and water temperature for all subjects participating in a given immersion. The thermal resistances ($1/h_1+1/h_2$) were determined by trial variations of $h_1$ which yielded the best computed fit to the time course of rectal temperatures for each subject using the best representation of metabolic heat production (see Methods). These results show that the maximum whole body insulation depends critically on body mass as well as on water temperature.

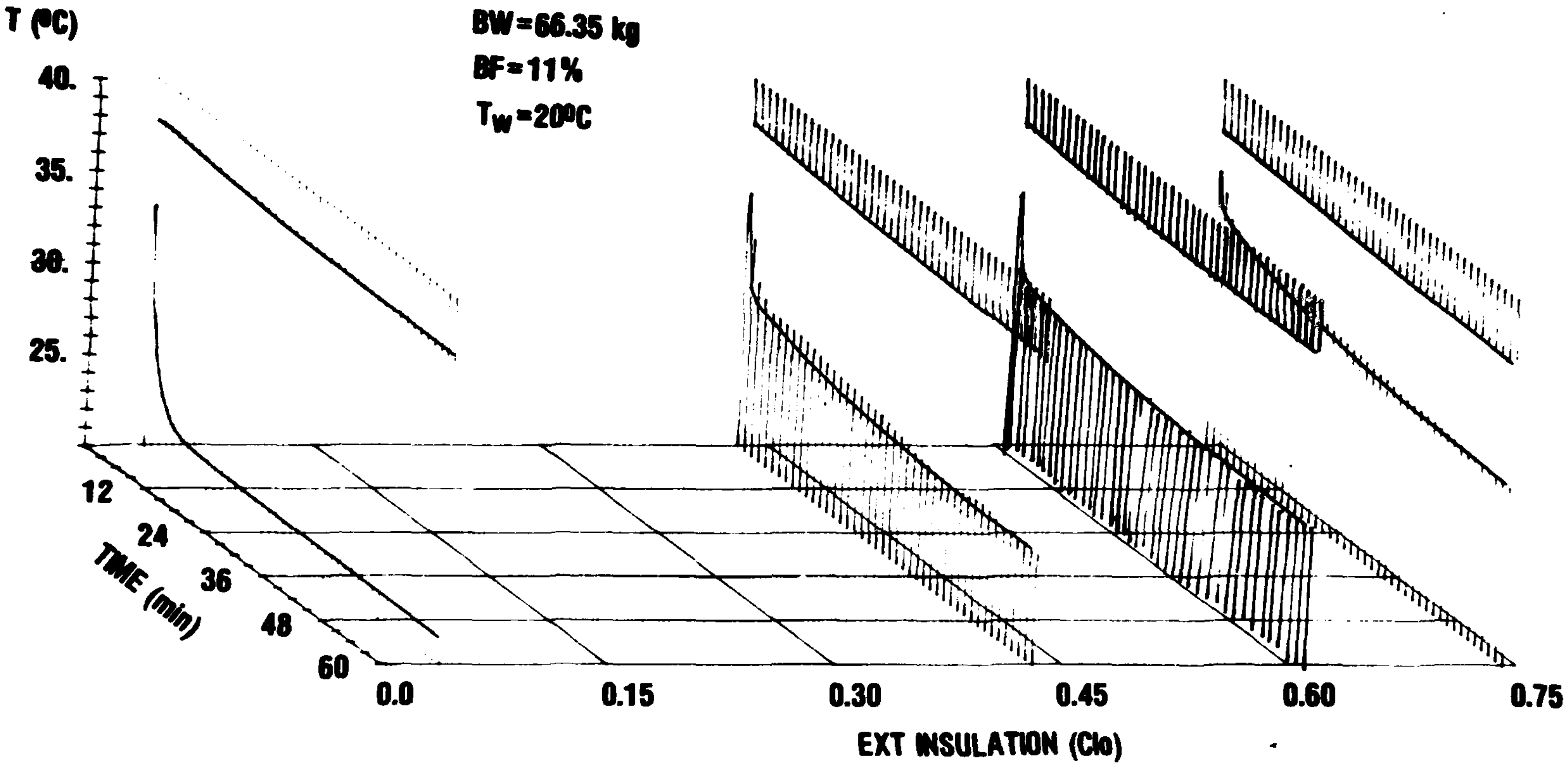



Figure 12A Experimental versus computed skin and rectal temperatures of a small, lean subject as functions of time and external insulation worn. Solid, continuous, line = computed temperatures. Vertical lines rising from the bottom grid terminate at points whose temperature component is the measured skin temperature; vertical lines descending from the top scale limit (40°C) terminate at points whose temperature component is the measured rectal temperature.

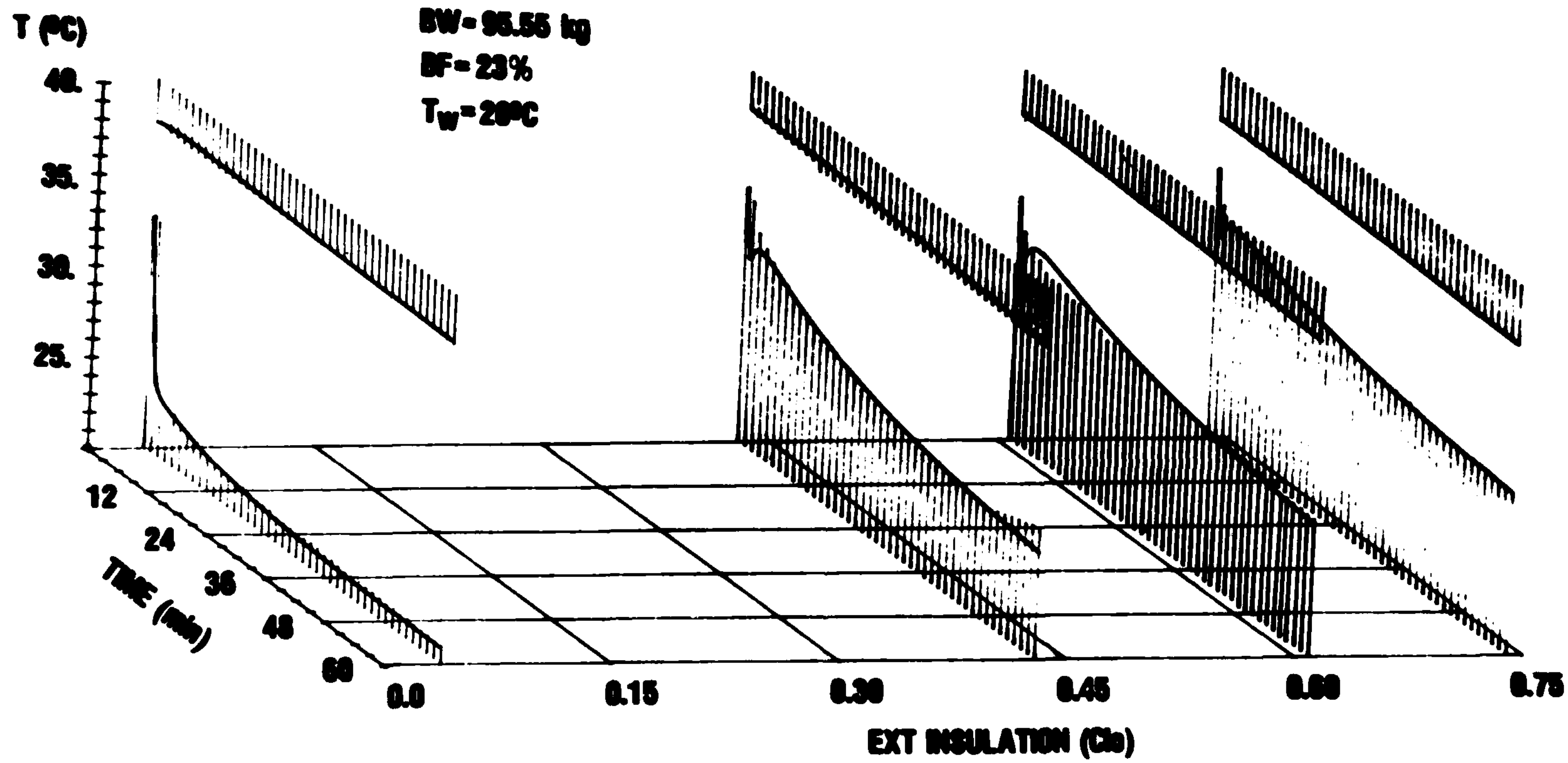



Figure 12B Experimental versus computed skin and rectal temperatures of a a heavy, fat subject as functions of time and external insulation worn. Solid, continuous, line = computed temperatures. Vertical lines rising from the bottom grid terminate at points whose temperature component is the measured skin temperature; vertical lines descending from the top scale limit (40°C) terminate at points whose temperature component is the measured rectal temperature.

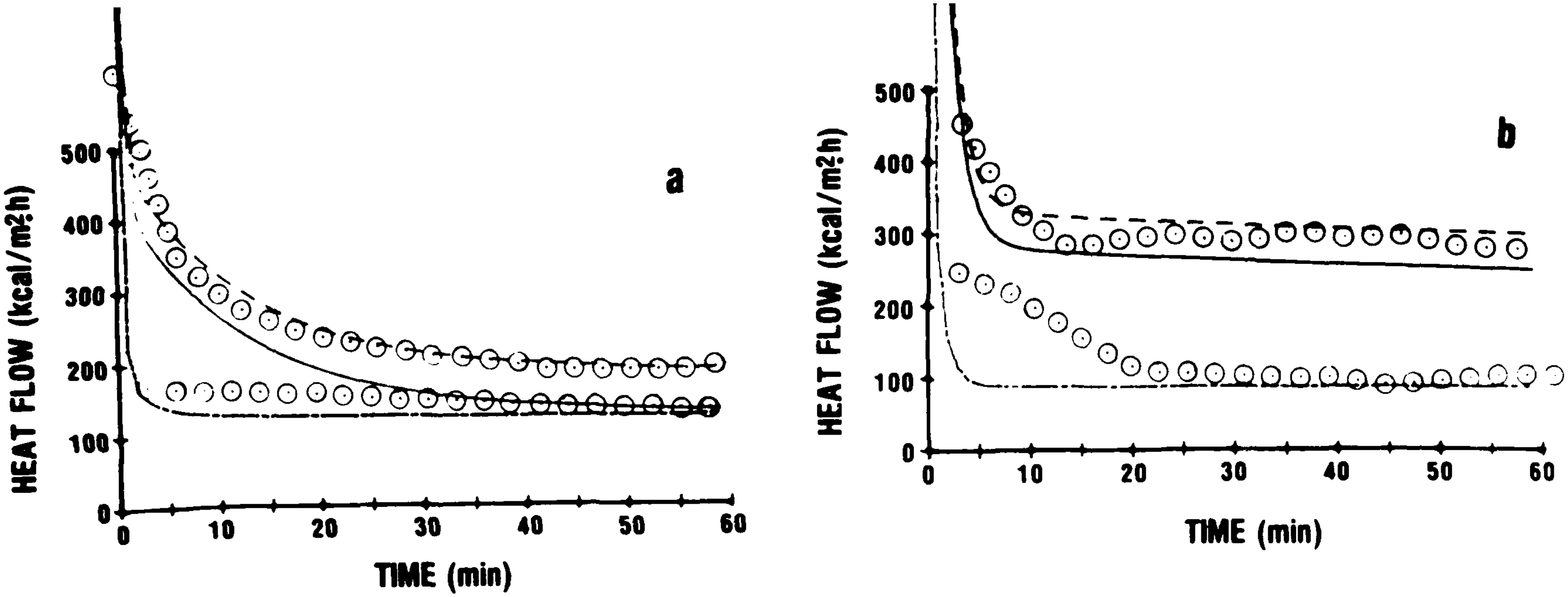



Figure 13 A comparison of the computed and measured mean heat flow for: a) heavy and b) lean subjects of Figure 12, immersed at 20°C.

circled points : measured surface heat flow for subjects both nude in water and clad in the 0.74 clo wetsuit.
solid line : computed surface heat flow for subjects nude in water.
speckled line computed surface heat flow for subjects clad in the 0.74 clo wet suits.
dashed line computed surface heat flow for subjects nude in water if the surface temperature under the heat flow sensors is elevated by 0.4°C to account for the intrinsic insulation of these sensors.

APPENDIX

THE COLD WATER IMMERSION PREDICTION MODEL (HP 9825 BASIC)

```
0: sfo 14
1: pen# 2
2: scl 0,24,0,24;xax 0,4,0,24;yax 0,2,0,20;stp
3: fxd 4
4: ent N;N)r28;ent S
5: dim X[0:N],Y[0:N],U[N,N],G[N,N],F[N],J[N,N,N],P[N,N,N]
6: dim L[0:7,0:7],A[-5:6],T[N+1],R[0:N+1],C[N],D[N]
7: dim I[N,N],H[0:N+1],Q[N],B[-5:2N],W[N]
8: dim M[N],V[N],S[N],N[N],Z[N],O[0:7,0:7],E[S,N]
9: dsp "ENTER MASS/AREA ith COMPARTMENT";wait 5000
10: for I=1 to N
11: enp M[I];next I
12: spc 2
13: dsp "ENTER SPECIFIC HEAT FOR ith COMPARTMENT";wait 5000
14: for I=1 to N
15: enp V[I];next I
16: spc 2
17: for I=1 to N
18: V[I]*M[I])S[I]
19: next I
20: dsp "ENTER THERMAL RESISTANCE FOR ith COMPARTMENT";wait 5000
21: for I=1 to N
22: enp R[I];next I
23: spc 2
24: dsp "ENTER SUBJECT SURFACE AREA,BODY FAT,&HEIGHT";wait 3000
25: enp H[3];enp H[4];enp H[0]
26: H[3]/(2*(*H[0]))H[1];H[1]-.0015)H[2];H[4]/(1-H[4]))H[4]
27: (H[2]^2-H[4]*1.19826*H[1]^2/(1+H[4]*1.19826))^.5)H[1]
28: 2*(*H[1]*H[0])H[1];2*(*H[2]*H[0])H[2]
29: spc 2
30: dsp "ENTER HEAT APPLIED TO ith COMPARTMENT";wait 5000
31: for I=1 to N
32: enp Q[I];Q[I])r(44+I);next I
33: spc 2
34: spc 1
35: dsp "ENTER INITIAL TEMP DISTRIBUTIONS FOR ith COMPARTMENT"
36: wait 5000
37: for I=1 to N
38: enp W[I];next I
39: spc 2
40: dsp "ENTER CORE DERIVED METABOLIC HEAT COEFFICIENTS";wait 5000
41: for I=1 to N
42: enp I[1,I];I[1,I]/H[1])O[1,I]
43: next I
44: spc 2
45: dsp "ENTER AMBIENT TEMP Ta";wait 3000
```

```
46: enp T[N+1]
47: for I=1 to N
48: 0→T[I];next I
49: dsp "ENTER DELTA TIME INCREMENT";wait 3000
50: enp r23;spc 2
51: jmp 19
52: "30":prt "set flag1;1 to change input;0 to terminate calculation"
53: stp
54: if flg1;jmp 2
55: gto "END"
56: prt "ENTER NEW INPUT STRING";stp ;r28→N
57: for I=0 to N+1
58: for J=0 to N+1
59: 0→O[I,J]
60: next J
61: next I
62: for I=1 to N
63: fmt f.0,f.4;wrt 16,"M[",I,"]=",M[I],"V[",I,"]=",V[I]
64: fmt f.0,f.4;wrt 16,"R[",I,"]=",R[I],"Q[",I,"]=",r(44+I);V[I]*M[I]→S[I]
65: fmt f.0,f.4;wrt 16,"H[",I,"]=",H[I],"W[",I,"]=",W[I]
66: fmt f.0,f.4;wrt 16,"I[1,",I,"]=",I[1,I],"I[3,",I,"]=",I[3,I]
67: I[1,I]/H[1]→O[1,I]
68: next I
69: prt "Ta=",T[N+1]
70: for I=1 to N;r(44+I)/H[I]→Q[I];next I
71: 10000→R[0];0→Q[4]→Q[5]
72: for I=1 to N
73: W[I]/S[I]+T[I+1]*(1/R[I])/S[I]→Z[I]
74: -1/(S[I]*R[I])-1/(S[I]*R[I-1])+O[I,I]/S[I]→O[I,I]
75: 1/(S[I]*R[I])+O[I,I+1]/S[I]→O[I,I+1]
76: 1/(S[I]*R[I-1])+O[I,I-1]/S[I]→O[I,I-1]
77: next I
78: for I=1 to N
79: for J=1 to N
80: O[I,J]→L[J,I]
81: next J
82: next I
83: L[1,1]+L[2,2]+L[3,3]→r8
84: L[4,4]+L[5,5]+L[6,6]→r9
85: L[3,4]*(L[1,1]+L[2,2])→r10
86: L[4,5]*L[5,3]-L[4,3]*L[5,5]-L[4,3]*L[6,6]→r11
87: L[1,1]*L[2,2]+L[1,1]*L[3,3]+L[2,2]*L[3,3]→r12
88: L[2,3]*L[3,2]+L[1,2]*L[2,1]+L[3,1]*L[1,3]-r12→r12
89: L[1,1]*L[2,2]*L[3,3]+L[1,2]*L[2,3]*L[3,1]→r13
90: L[1,3]*L[2,1]*L[3,2]+r13-L[1,1]*L[2,3]*L[3,2]→r13
```

```
91: r13-L[1,2]*L[2,1]*L[3,3]-L[3,1]*L[1,3]*L[2,2]→r13
92: L[1,3]*L[3,4]→r14
93: L[4,4]*L[5,5]+L[4,4]*L[6,6]+L[5,5]*L[6,6]-L[5,6]*L[6,5]→r15
94: r15-L[4,5]*L[5,4]→r15
95: L[4,4]*L[5,5]*L[6,6]-L[4,4]*L[5,6]*L[6,5]-L[4,5]*L[5,4]*L[6,6]→r16
96: L[1,2]*L[2,1]*L[3,4]-L[3,4]*L[1,1]*L[2,2]→r17
97: L[4,3]*L[5,5]*L[6,6]-L[4,3]*L[6,5]*L[5,6]→r18
98: r18-L[4,5]*L[5,3]*L[6,6]+L[4,5]*L[6,3]*L[5,6]→r18
99: L[1,3]*L[2,2]*L[3,4]-L[1,2]*L[2,3]*L[3,4]→r19
100: L[4,5]*L[5,1]-L[4,1]*L[5,5]-L[4,1]*L[6,6]→r20
101: L[4,1]*L[5,5]*L[6,6]-L[4,1]*L[6,5]*L[5,6]-L[4,5]*L[5,1]*L[6,6]→r21
102: r21+L[4,5]*L[6,1]*L[5,6]→r21
103: 1→A[N]
104: -(r8+r9)→A[N-1]→r37
105: -r12+r8*r9+r15-L[3,4]*L[4,3]→A[N-2]→r38
106: -(r13-r9*r12+r15*r8+r16-r10*L[4,3]+r11*L[3,4]+r14*L[4,1])→W
107: W→A[N-3]→r39
108: r13*r9+r8*r16-r12*r15+r17*L[4,3]+r10*r11→W
109: W-L[3,4]*r18+L[4,1]*r19-r20*r14→A[N-4]→r40
110: -(r13*r15-r16*r12-r17*r11-r10*r18-r20*r19+r14*r21)→A[N-5]→r41
111: r16*r13+r17*r18+r19*r21→A[N-6]→r42
112: 0→B[N]→B[N-1]→B[N-2]→B[N-3]→B[N-4]→B[N-5]→B[N-6]
113: for I=0 to N
114: fmt f.0,f.4;wrt 16,"A[",I,"]=",A[I]
115: next I
116: gsb "ROOT"
117: r28→N
118: for I=1 to N
119: C[I]→A[I];next I
120: gsb "ISORT"
121: 1→J
122: prt "ROOT NO.",J,"A=",A[J];jmp (J+1→J)>N
123: for J=1 to N
124: 1→U[N,J]→P
125: 0→W
126: 0→O[N,N+1]
127: for I=1 to N-1
128: if N-I=2;jmp 5
129: ((-O[N+1-I,N+1-I]+A[J])*P-O[N+1-I,N+2-I]*W)/O[N+1-I,N-I]→P→U[N-I,J]
130: U[N+1-I,J]→W
131: next I
132: jmp 8
133: 0→r22
134: N→K
135: if N>3;-O[3,K]*U[K,J]+r22→r22;jmp (K-1→K)<4
```

```
if N>2;r22+(O[3,1]*O[2,3]/O[2,1]-(O[3,3]-A[J]))*U[3,J]→r22
r22/(O[3,2]-O[3,1]*(O[2,2]-A[J])/O[2,1])→U[2,J]→P
U[3,J]→W
jmp -8
(O[1,1]-A[J])*U[1,J]→W
for I=2 to N
W+O[1,I]*U[I,J]→W
next I
if W=0;jmp 2
dsp "NO AGREEMENT J=",J;stp
next J
gsb "MATS"
gsb "PROD"
gto "PLOT"
"ROOT":cfg 2;1→X[0]→Y[1]→Y;.1→X[1]→X;0→L→Y[0]
1→X[0]→Y[1]→Y;.1→X[1]→X;0→L→Y[0]
gsb "SILJAK"
F→G;0→K→P→Q→M;L+1→L
if (K+1→K)>N;PP+QQ→Z;gto +3
K(A[K]X[K-1]-B[K]Y[K-1])+P→P
K(A[K]Y[K-1]+B[K]X[K-1])+Q→Q;gto -2
-(UP+VQ)/Z→D;(UQ-VP)/Z→E
M+1→M
X+D→X[1];Y+E→Y[1]
gsb "SILJAK"
if F>=G;gto +4
if abs(D)<1e-8 and abs(E)<1e-8;gto +13
if L>50;gto +4
X[1]→X;Y[1]→Y;gto -11
if M<=10;D/4→D;E/4→E;gto -7
if abs(U)<=1e-7 and abs(V)<=1e-7;gto +9
sfg 2;dsp "NO CONVERGENCE";wait 3000;r28→N;gto "ROOTS"
"SILJAK":X[1]X[1]+Y[1]Y[1]→Z;-1→K;2X[1]→T
if (K+1→K)>N-2;0→U→V;-1→K;gto +3
TX[K+1]-ZX[K]→X[K+2]
TY[K+1]-ZY[K]→Y[K+2];gto -2
if (K+1→K)>N;UU+VV→F;ret
A[K]X[K]-B[K]Y[K]+U→U
A[K]Y[K]+B[K]X[K]+V→V;gto -2
X[1]→C[N];Y[1]→D[N]
A[N]→A;B[N]→B;0→A[N]→B[N];N→K;X[1]→X;Y[1]→Y
if (K-1→K)<0;gto +4
A[K]→C;B[K]→D
A+X(A[K+1]→U)-Y(B[K+1]→V)→A[K]
B+XV+YU→B[K];C→A;D→B;gto -3
```

```
181: if (N-1]N)#1;gto -30
182: -((A[0]]A)(A[1]]U)+(B[0]]B)(B[1]]V))/(UU+VV)T)]C[1]
183: (AV-UB)/T]D[1];ret
184: "ISORT":
185: for I=1 to N-1
186: for J=1 to N-I
187: if A[I]<=A[I+J];A[I]]P;A[I+J]]A[I];P]A[I+J]
188: next J
189: next I
190: ret
191: "MATS":
192: inv U]G,D
193: mat G*W]C
194: ret
195: "PROD":
196: for I=1 to N
197: for J=1 to N
198: for K=1 to N
199: Z[K]*U[J,I]*G[I,K]]P[I,J,K]
200: U[I,J]*G[J,K]]J[J,I,K]
201: next K
202: next J
203: next I
204: ret
205: "PLOT":0]r7
206: for P=1 to N
207: -r23]T
208: for R=1 to S
209: T+r23]T
210: for Q=1 to N
211: A[Q]*T]rQ
212: next Q
213: 0]r29
214: for M=1 to N
215: if rM<-200;0]r30;jmp 2
216: exp(rM)]r30
217: for I=1 to N
218: if rM-rI>0;0]r31;jmp 3
219: if rM-rI<-200;0]r31;jmp 2
220: exp(rM-rI)]r31
221: for J=1 to N
222: for L=1 to N
223: J[M,P,J]*P[I,J,L]/A[I]*(r30-r31)+r29]F[P]]r29
224: next L
225: next J
```

```
226: next I
227: next M
228: for I=1 to N
229: if rI<-200;jmp 2
230: F[P]+C[I]*U[P,I]*exp(rI)→F[P]
231: next I
232: F[P]→E[R,P]
233: plt T*24,F[P]-20
234: next R
235: pen
236: next P
237: pen
238: stp
239: scl 0,24,0,50;xax 0,4,0,24;yax 0,10,0,50
240: -r23→T
241: for R=1 to S
242: T+r23→T
243: (E[R,3]-E[R,4])/(10*R[3])→O
244: plt T*24,O
245: next R
246: pen
247: gto "30"
248: "END":end
249: "ROOTS":for I=1 to N
250: enp "LOW BOUND?",r32
251: enp "UPPER BOUND?",r33
252: enp "SEARCH INC?",r34
253: prt "ROOTS ARE";spc
254: if r32>r33;dsp "ILLEGAL BOUNDS";jmp -4
255: r32→X;gsb "EVAL"
256: Y→F
257: if (r32+r34→r32)>r33;jmp 18
258: r32→X;gsb "EVAL"
259: if (FY→Z)>0;gto -3
260: if Z<0;gto +3
```

```
261: if F=0;r32-r34→X;F→Y
262: prt "X=",X,"F(X)=",Y;spc ;r32+r34→r32;1e-12→r36;gto +9
263: r32-r34→L;r32→R;0→C
264: (L+R)/2→X;gsb "EVAL"
265: if (C+1→C)>50;gto +5
266: if abs(Y)<1e-7;0→Z;gto +2
267: if (FY→Z)>0;X→L;gto -3
268: if Z=0;prt "X=",X,"F(X)=",Y;spc ;R-L→r36;X→C[I];gto +3
269: X→R;gto -5
270: prt "X BETWEEN",L,R,"F(X)=",Y;spc ;R-L→r36;R→C[I]
271: spc ;prt "ACCURATE TO",r36;spc ;gto -16
272: "EVAL":
273: r42+r41*X^(N-5)+r40*X^(N-4)+r39*X^(N-3)+r38*X^(N-2)+r37*X^(N-1)+X^N→Y
274: ret
275: next I
276: gto 102
```

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